Time-reversal symmetric topological superconductivity in Machida-Shibata lattices

Thore Posske, Universität Hamburg, Germany
ICSM 2025, Fethiye, Turkiye

Time-reversal symmetric topological superconductivity in Machida-Shibata lattices

Ioannis Ioannidis,[1,2] Ching-Kai Chiu,[3] and Thore Posske[1,2]

[1] *I. Institute for Theoretical Physics, University of Hamburg, D-22607 Hamburg, Germany*
[2] *Centre for Ultrafast Imaging, Luruper Chaussee 149, D-22761 Hamburg, Germany*
[3] *RIKEN Interdisciplinary Theoretical and Mathematical Sciences (iTHEMS), Wako, Saitama 351-0198, Japan*
(Dated: April 14, 2025)

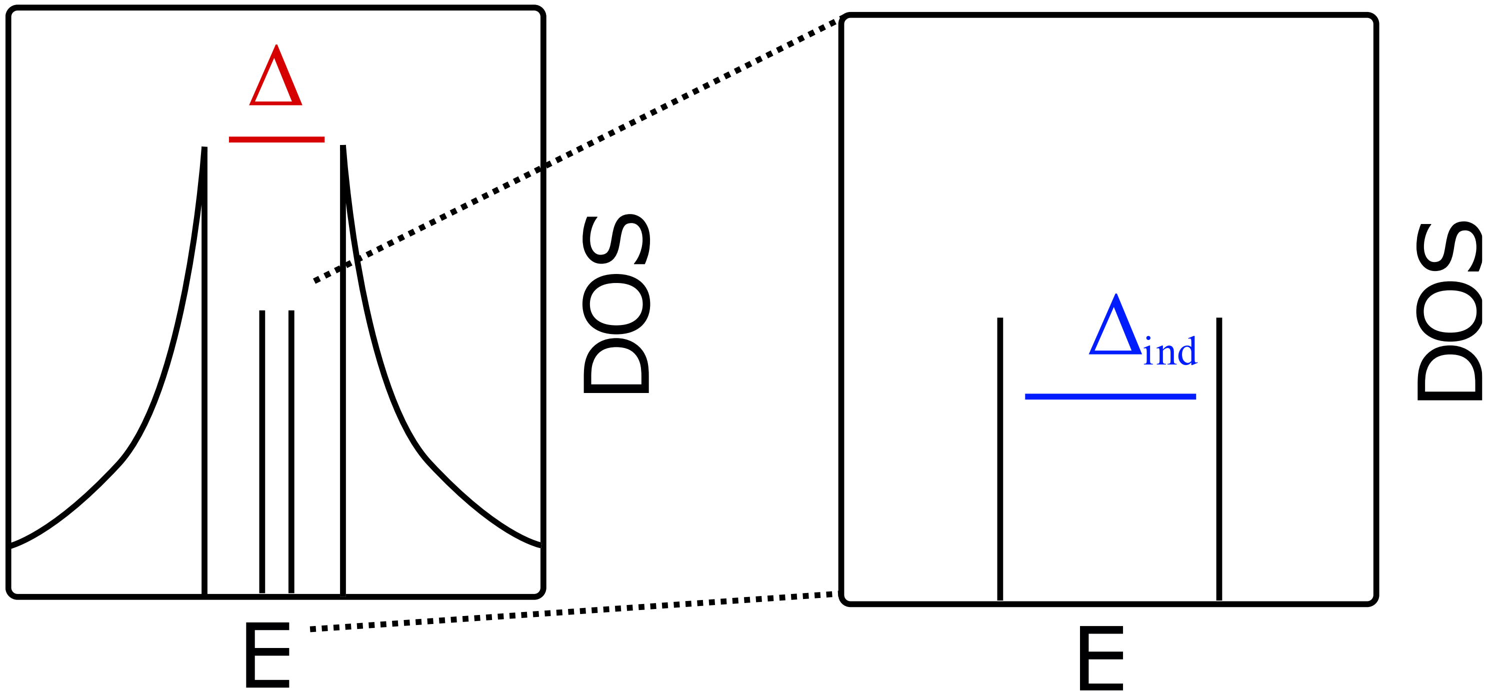
arXiv:2504.08674 (2025) www.posske.de

Summary

- **Nonmagnetic in-gap state** on superconductors: **Machida-Shibata state**
- **Hybridizing lattices of Machida-Shibata states** accommodates **topological superconductor**, **class DIII**
- Small spin orbit coupling creates topologically nontrivial phase
- **Kramers degenerate Majorana bound states** (1D) and dispersive **chiral Majorana edge channels** (2D)
- Exact compensation of singlet and triplet superconductivity in parts of the Brillouin zone

Motivation

Low-energy (in-gap) electronic states in superconductors

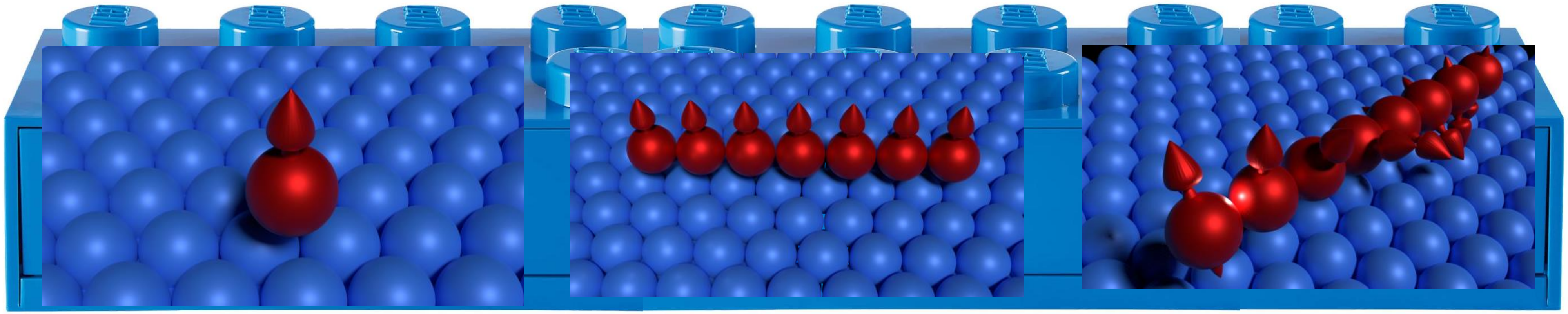


Lego kit of in-gap states for synthesizing 2D quantum matter

Yu-Shiba-Rusinov states

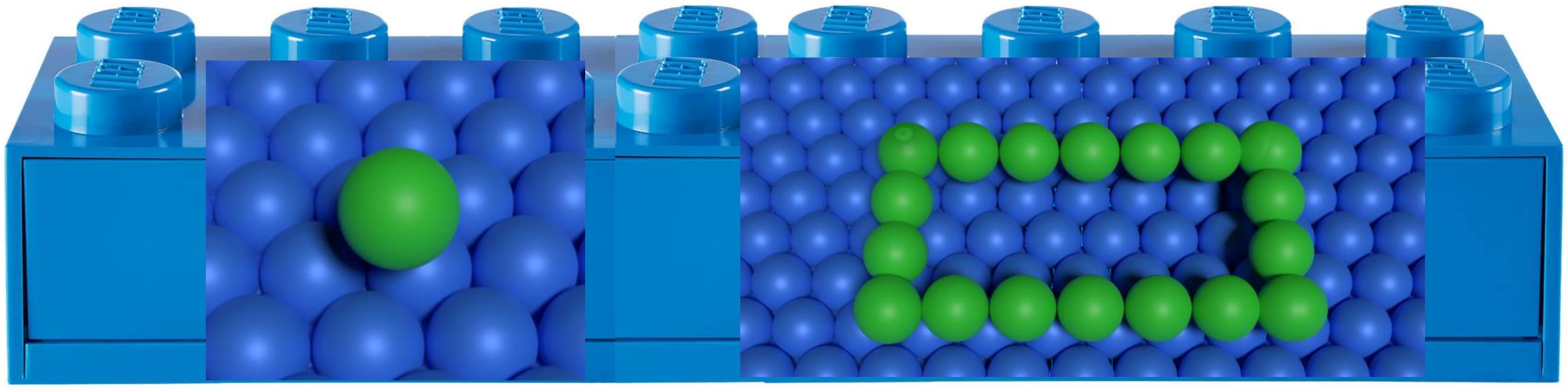
Shiba chains
Precursors of Majorana modes

Helical magnetism
Majorana modes

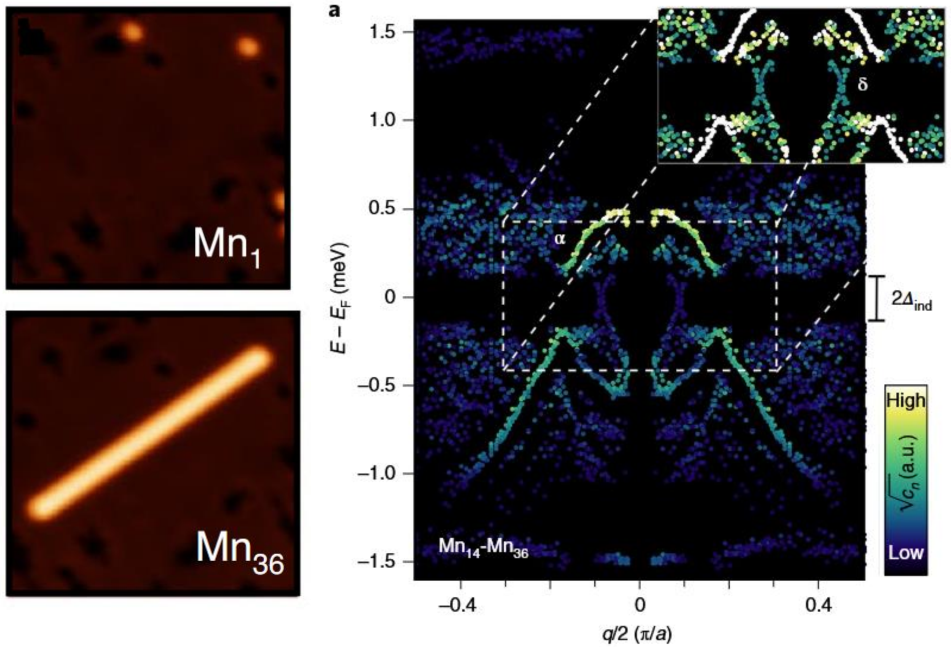


Nonmagnetic adatoms

Machida-Shibata states

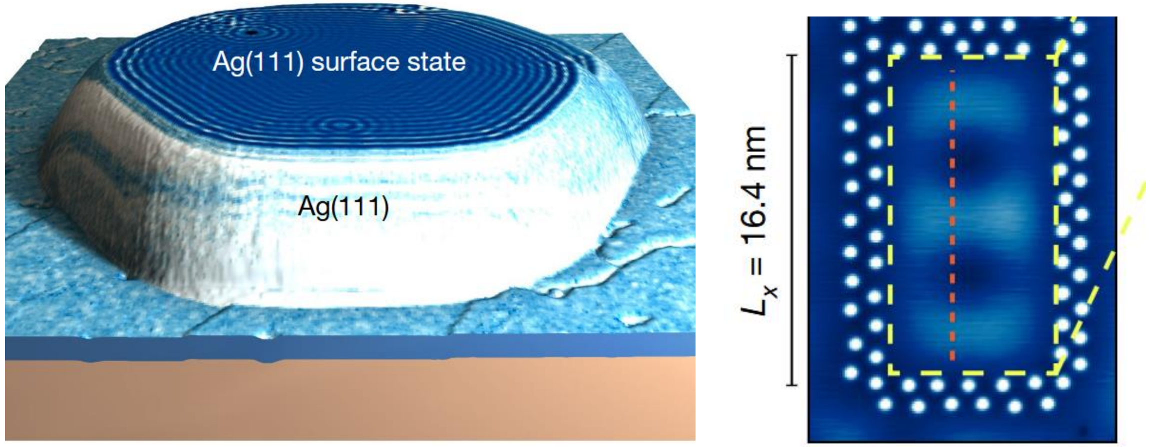


Motivation



Magnetic adatoms
Yu-Shiba-Rusinov states
hybridization ⇒ top. YSR bands

Schneider et al., Nature Physics **17**, 943 (2021)



Nonmagnetic adatoms
Machida-Shibata states
hybridization ⇒ ?

Schneider et al., Nature **621**, 60 (2023)

Motivation

RAPID COMMUNICATION

No-go theorem for a time-reversal invariant topological phase in noninteracting systems coupled to conventional superconductors

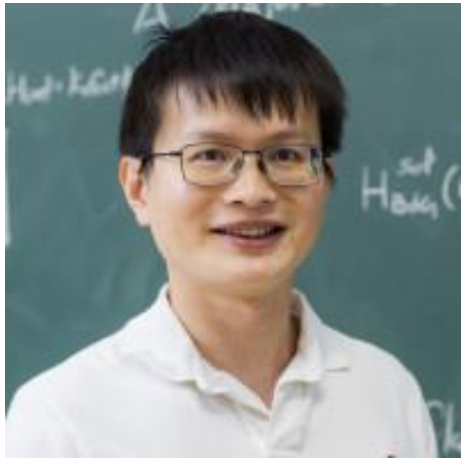
Arbel Haim[1], Erez Berg[1], Karsten Flensberg[2], and Yuval Oreg[1]

Show more

Phys. Rev. B **94**, 161110(R) – **Published 10 October, 2016**

DOI: https://doi.org/10.1103/PhysRevB.94.161110

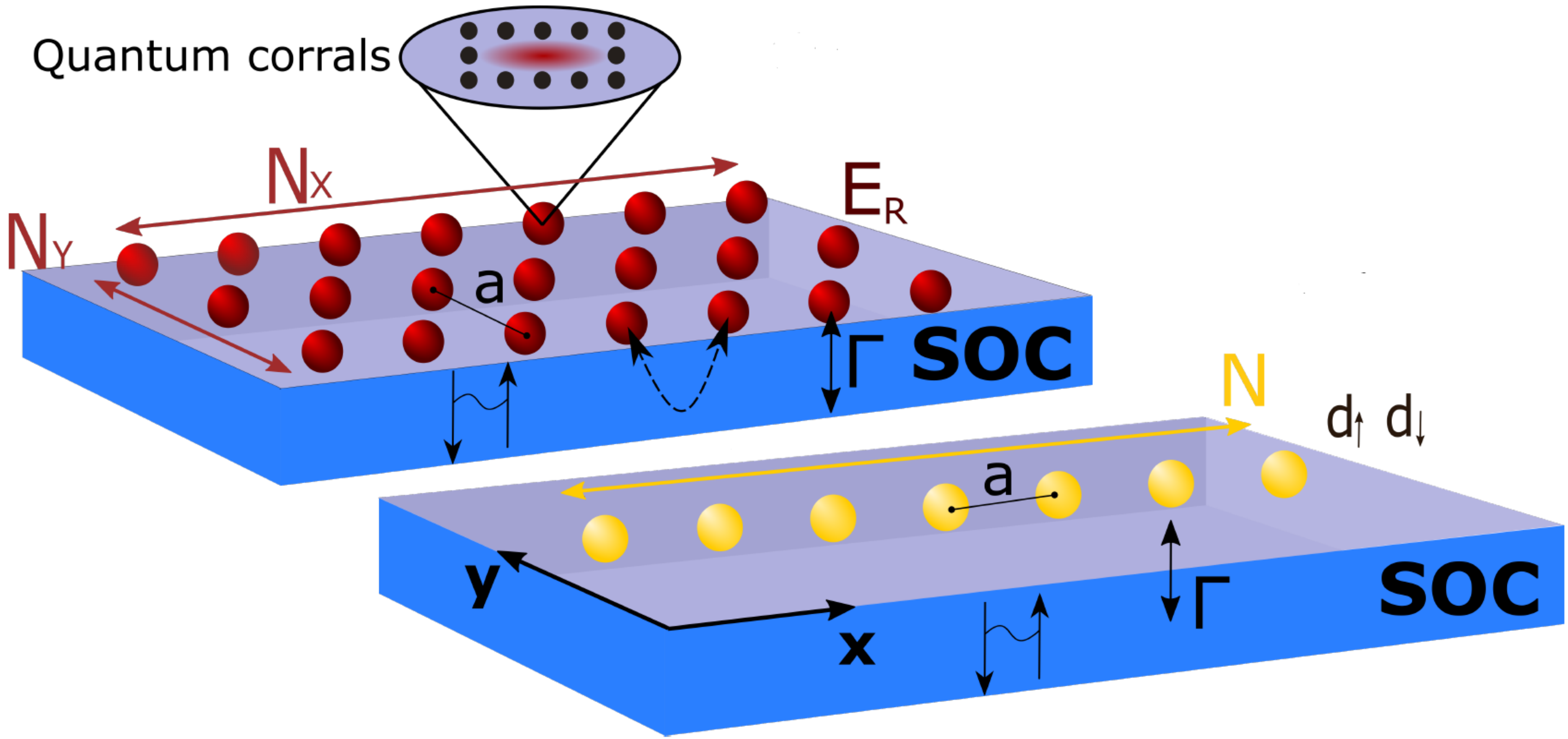
People



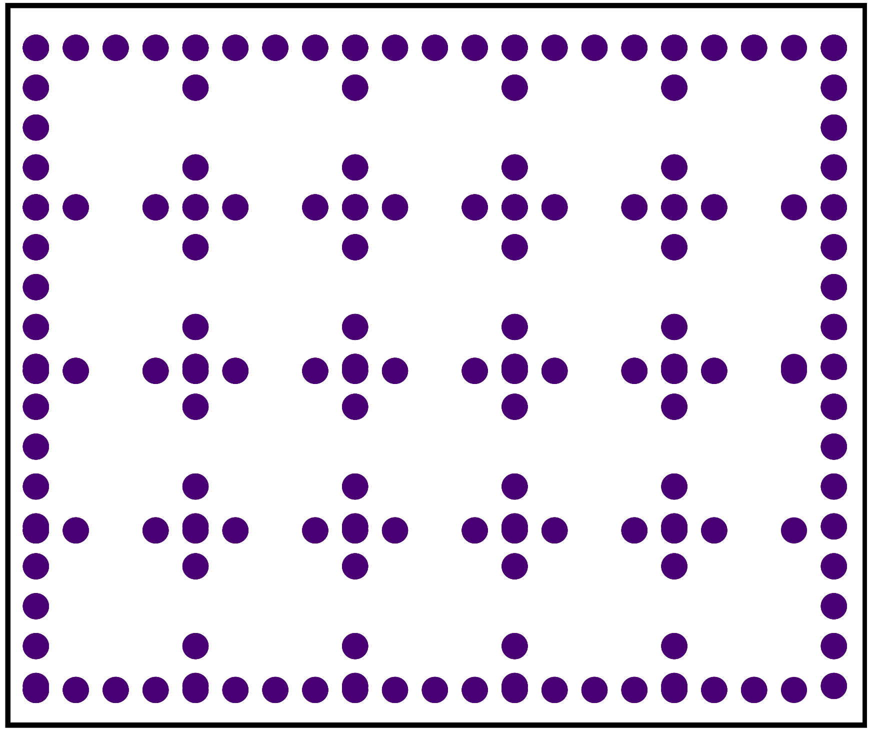
Ioannis Ioannidis
U Hamburg

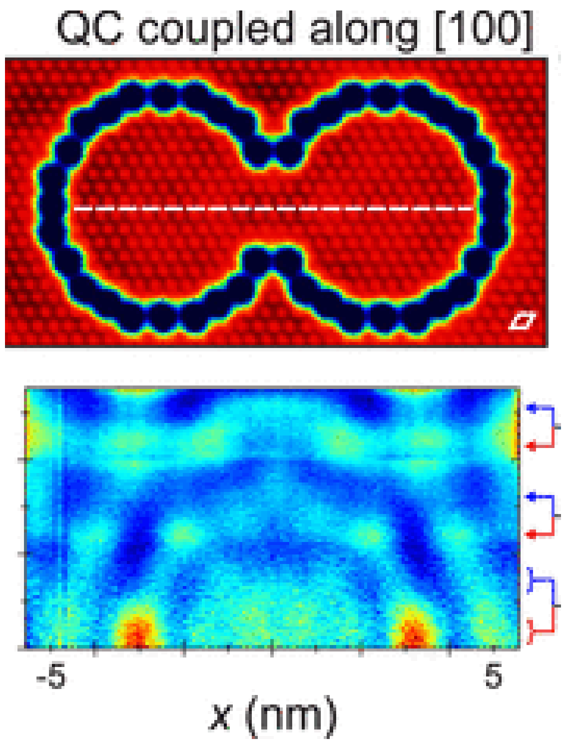
Ching-Kai Chiu
RIKEN

Machida-Shibata lattices

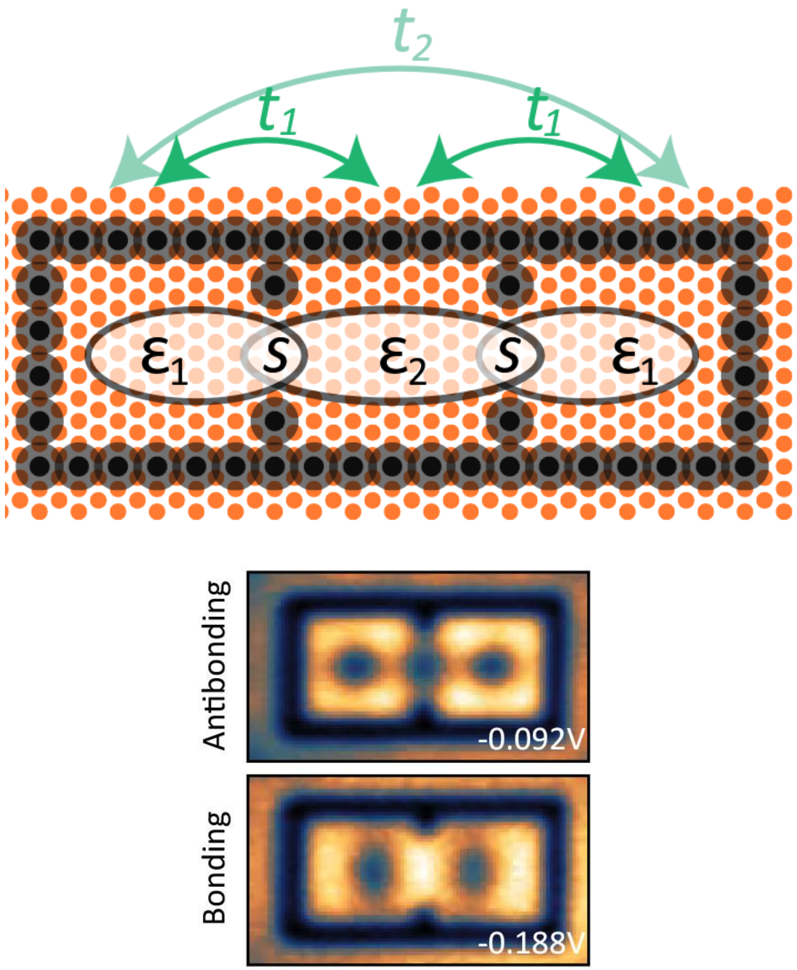


Machida Shibata lattices, proposal





Jolie … Khajetoorians et al.
ACS Nano, 16, 4876 (2022)



Freeney et al.,
SciPost Phys. 9, 085 (2020)

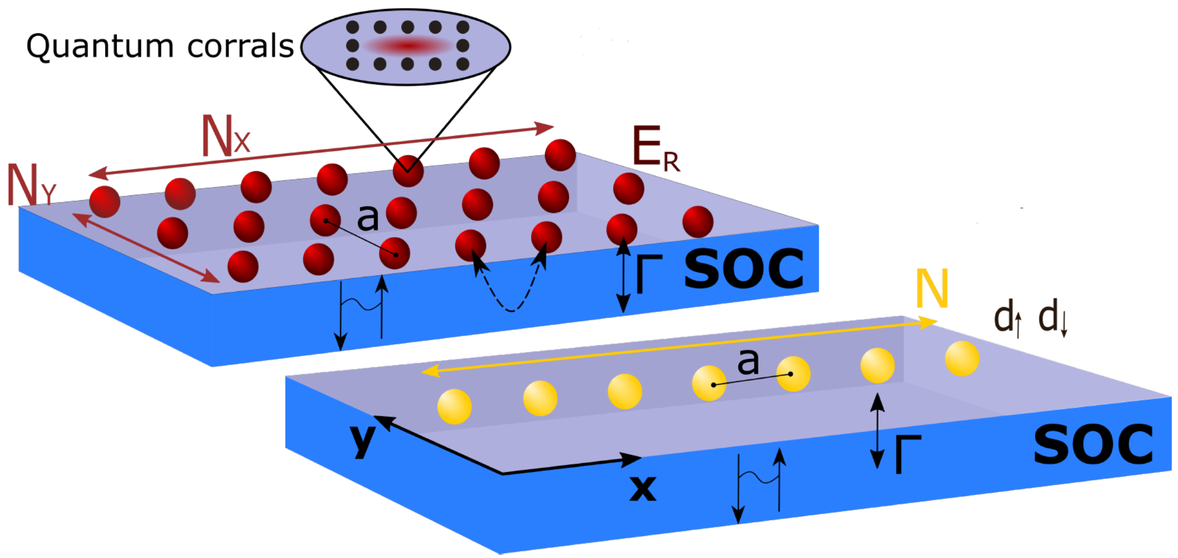
Symmetry class

- Time-reversal symmetry T (no magnetic field)
- Particle-hole symmetry C (superconductor)
- Chiral symmetry ($S = \mathrm{T} \cdot \mathrm{C}$)

	Symmetry			Dimension							
AZ	T	C	S	1	2	3	4	5	6	7	8
A	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AI	1	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
C	0	-1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
CI	1	-1	1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0

Hamiltonian

$$\hat{H} = \hat{H}_{\mathrm{MS}} + \hat{H}_{\mathrm{SC}} + \hat{H}_{\mathrm{T}}$$

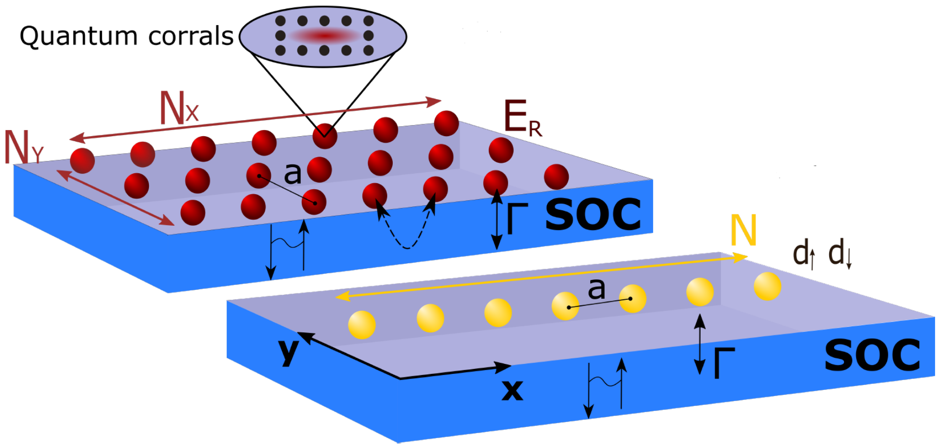


$$\hat{H}_{\mathrm{MS}} = E_{\mathrm{R}} \sum_{\sigma,j} d^{\dagger}_{\sigma,j} d_{\sigma,j}$$

$$\hat{H}_{\mathrm{T}} = V \sum_{\mathbf{k},\sigma,j} \left(e^{i\mathbf{k}\mathbf{R}_j} c^{\dagger}_{\mathbf{k},\sigma} d_{\sigma,j} + h.c \right)$$

$$\hat{H}_{\mathrm{SC}} = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k},\sigma} c_{\mathbf{k},\sigma} + \lambda \sum_{\mathbf{k}} |\mathbf{k}| \left(i e^{-i\theta(\mathbf{k})} c^{\dagger}_{\mathbf{k},\uparrow} c_{\mathbf{k},\downarrow} + h.c \right) - \Delta \sum_{\mathbf{k}} \left(c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow} + h.c \right)$$

Effective Hamiltonian



$$H_{\text{eff}} = \begin{pmatrix} h_{i,j} & \Delta_{i,j} \\ \Delta_{i,j}^{\dagger} & -\sigma_z h_{i,j} \sigma_z \end{pmatrix}$$

$$\hat{h}_{i,j} = \begin{pmatrix} h_{i,j}^{N} & h_{i,j}^{F} \\ h_{i,j}^{F\dagger} & h_{i,j}^{N} \end{pmatrix} \qquad \hat{\Delta}_{i,j} = \begin{pmatrix} \Delta_{i,j}^{S} & \Delta_{i,j}^{T} \\ \Delta_{i,j}^{T*} & -\Delta_{i,j}^{S} \end{pmatrix}$$

$$\begin{pmatrix} d_{\uparrow,j}^{\dagger} & d_{\downarrow,j}^{\dagger} & d_{\downarrow,j} & d_{\uparrow,j} \end{pmatrix}^{\dagger}$$

Effective parameters

$$h^N_{i,j} = E_{\mathrm{R}}\delta_{i,j} + (1-\delta_{i,j})\mathrm{Im}\left(w^e_{i,j}\right), \quad \Delta^S_{i,j} = -\Gamma\delta_{i,j} - (1-\delta_{i,j})\mathrm{Re}\left(w^e_{i,j}\right),$$

$$h^F_{i,j} = (1-\delta_{i,j})e^{-i\phi_{i,j}}\mathrm{Re}\left(w^o_{i,j}\right), \qquad \Delta^T_{i,j} = -(1-\delta_{i,j})e^{-i\phi_{i,j}}\mathrm{Im}\left(w^o_{i,j}\right),$$

$$w^o_{i,j} = \sum_{\mu=\pm} \frac{\mu\Gamma_\mu}{2}\left(iJ_1\left[x^\mu_{i,j}\right] + H_{-1}\left[x^\mu_{i,j}\right]\right)$$

$$w^e_{i,j} = \sum_{\mu=\pm} \frac{\Gamma_\mu}{2}\left(J_0\left[x^\mu_{i,j}\right] + iH_0\left[x^\mu_{i,j}\right]\right)$$

$$\Gamma_\pm = \pi\nu_\pm |V|^2 \qquad x^\pm_{j,m} = \left(k_{F_\pm} + i\xi^{-1}\right)|\mathbf{R}_j - \mathbf{R}_m|$$

Bessel and Struwe functions

Bessels differential equation

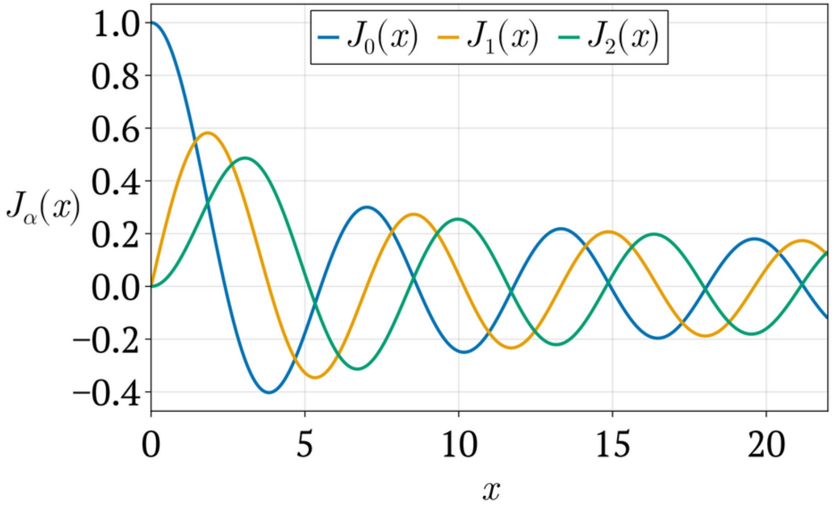
$$\left(x^2\partial_x^2 + x\partial_x + (x^2 - \alpha^2)\right)y \sim x\text{^}\alpha$$

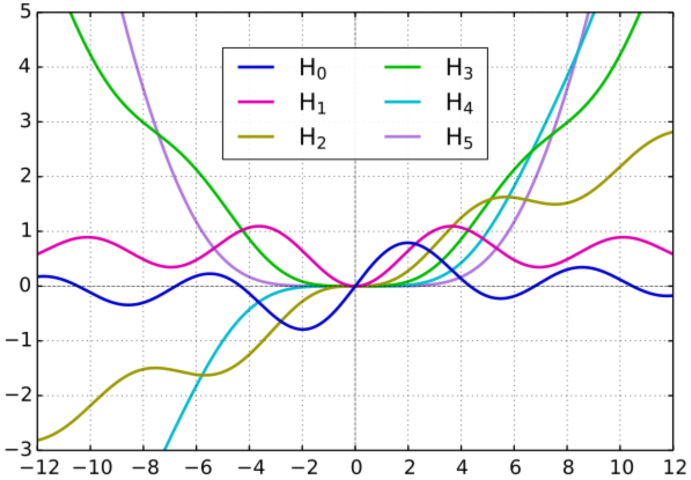
- Bessel function of first kind (hom. sol.)

$$J_\alpha(x) = \frac{1}{\pi} Re\left(\int_0^\pi e^{i(\alpha\tau - x\sin(\tau)}\, d\tau\right)$$

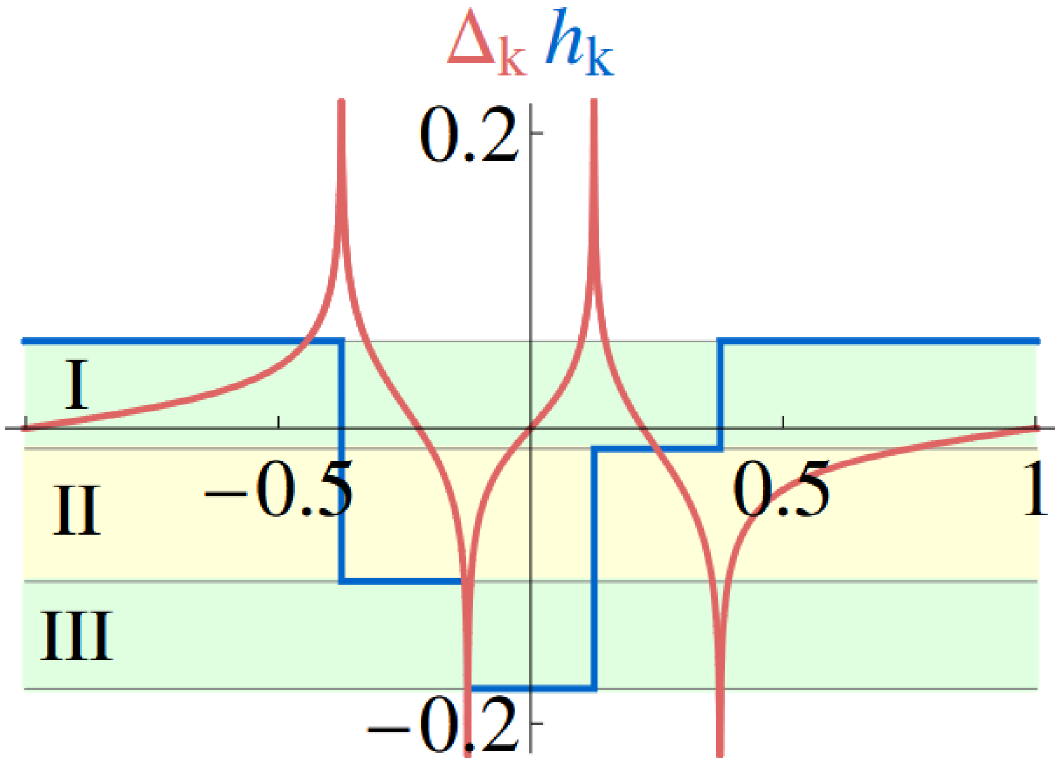
- Struve function (inhomogeneous solution)

$$H_\alpha\,(x) \sim \frac{1}{\pi}\int_0^{\pi/2} \sin(x\sin(\tau))\cos(\tau)^{2\alpha}\, d\tau$$





Effective parameters in momentum space example: YSR bands



Pientka et al., PRB **88**, 155420 (2013)

Solution – selected details

Green's function equations of motion

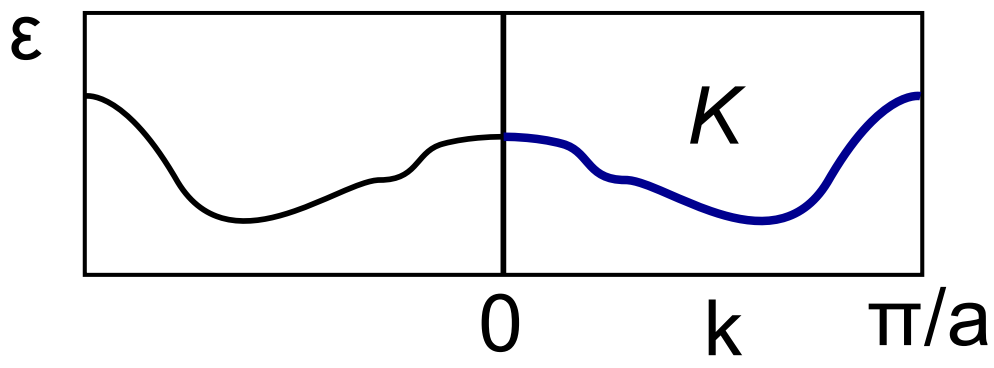
$$(E - \hat{H}_{\text{eff}}(E))\check{G} = \hat{1}$$

Gap closure condition in 1D

$$\Delta^S(p_0) = \pm\, \mathrm{Im}\{\Delta^T(p_0)\}$$
$$h^N(p_0) = \mp\, \mathrm{Im}\{h^F(p_0)\}$$

https://arxiv.org/abs/2504.08674

Topological analysis



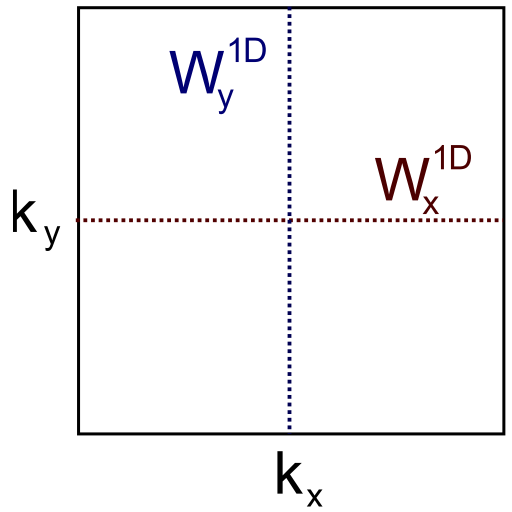
1D

- Class DIII topological invariant $$W^{1D} = \det(K)\frac{\mathrm{Pf}(\theta_0)}{\mathrm{Pf}(\theta_\pi)}$$

- With Kato propagator K from $k = 0$ to $k = \pi$
  θ_k represents time reversal symmetry.

2D

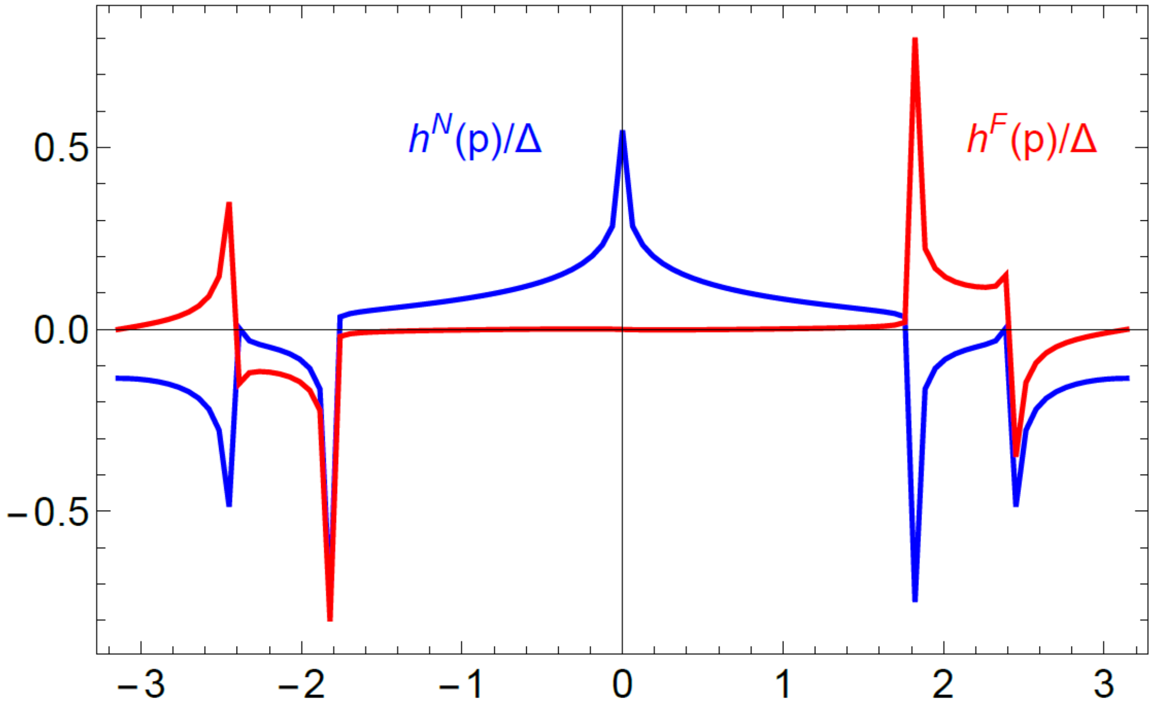
$$W^{2D} = W^{1D}_{k_y \to 0} \times W^{1D}_{k_x \to 0}$$

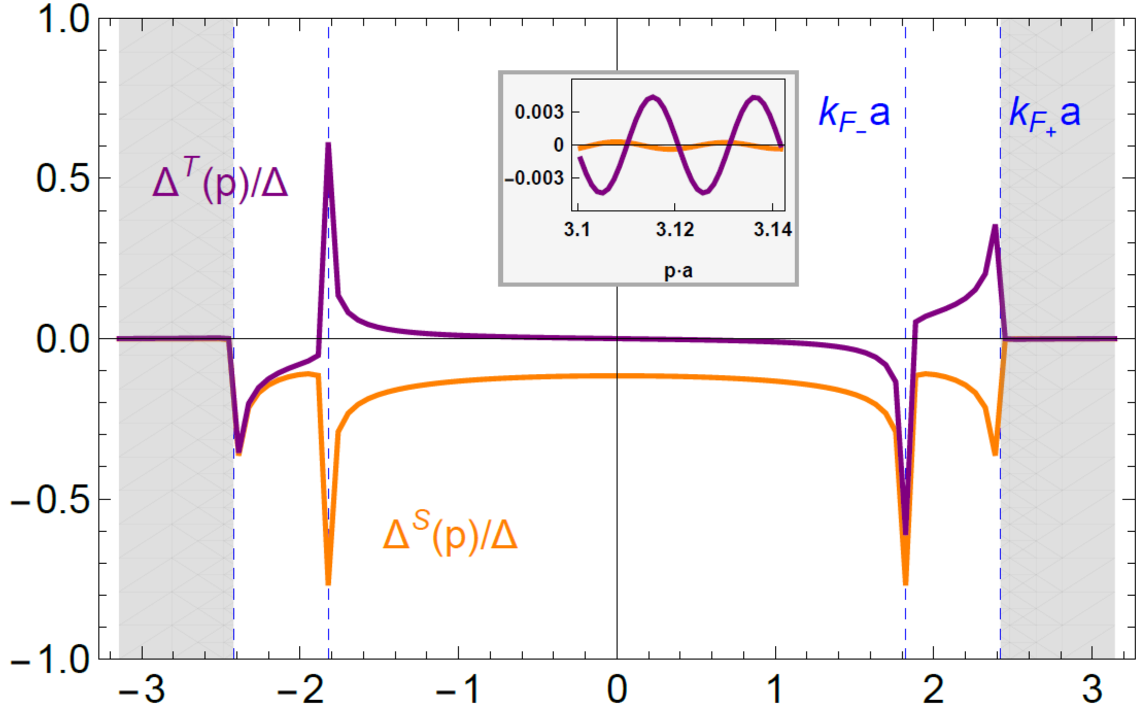


Ardonne, Budich Phys. Rev. **B** 88, 134523 (2013)

Haim, Berg, Flensberg, Oreg, Phys. Rev. B **94**, 161110(R) **(2016)**

Effective parameters in momentum space

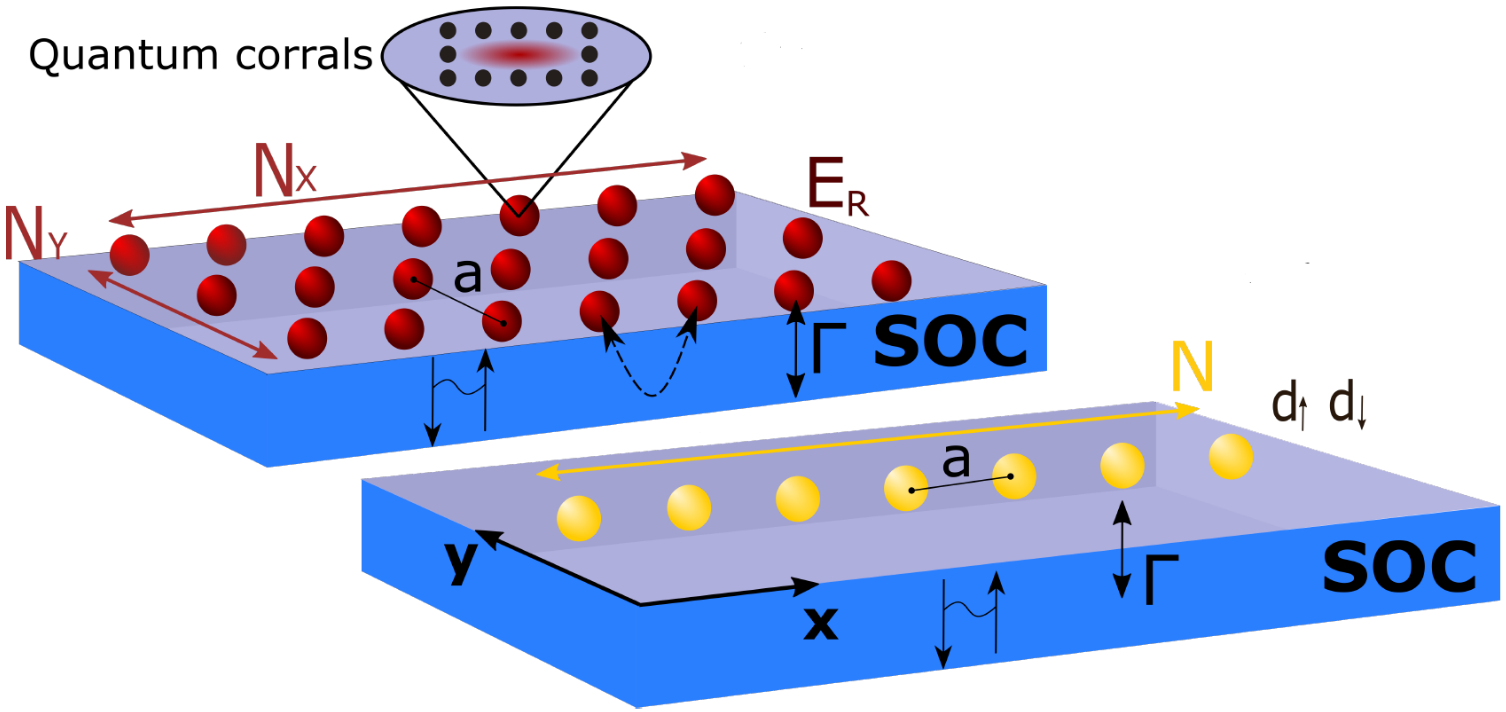


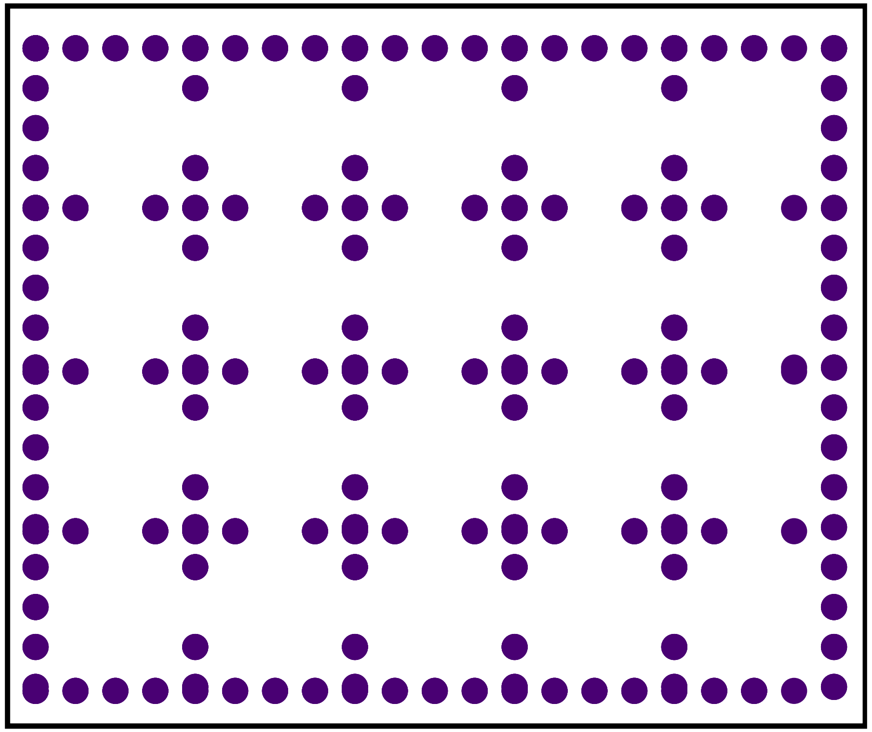


Problem: For topological superconductivity triplet > singlet needed

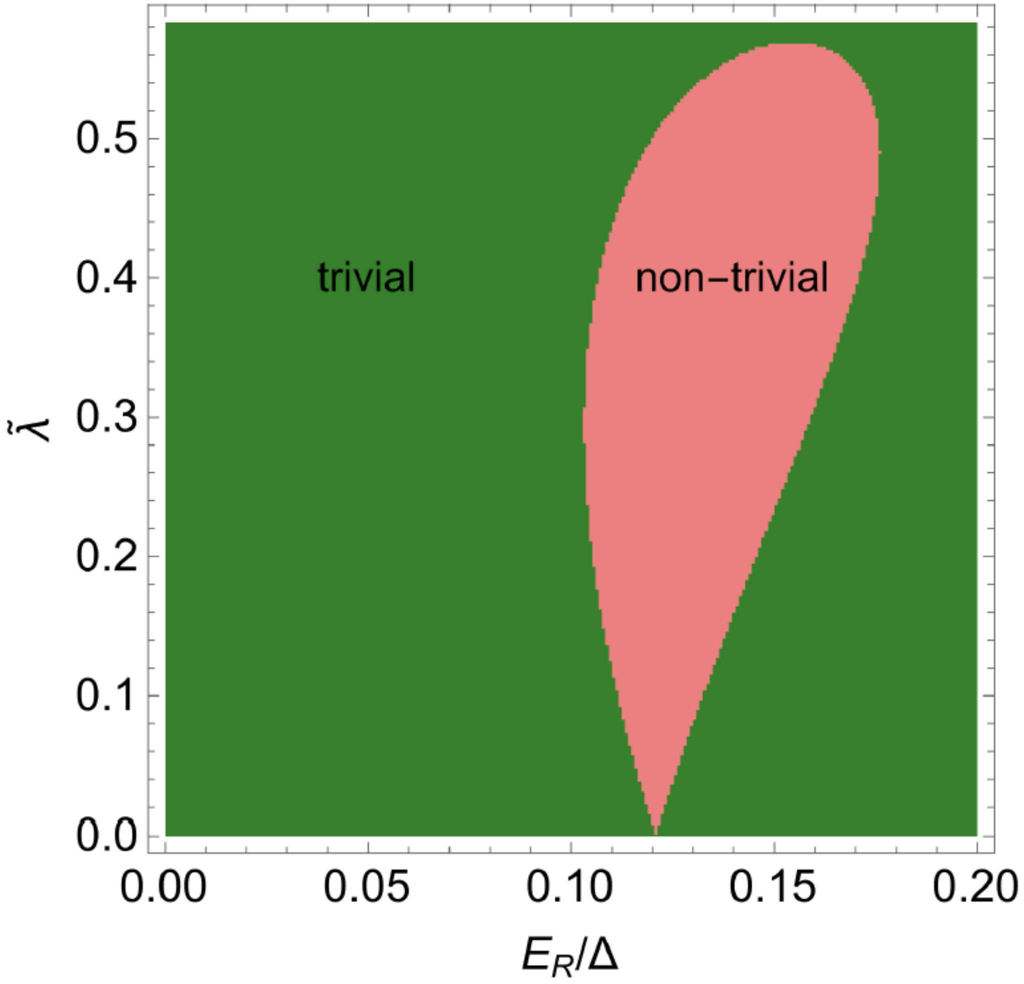
Solution 1 – Nearest neighbor cutoff

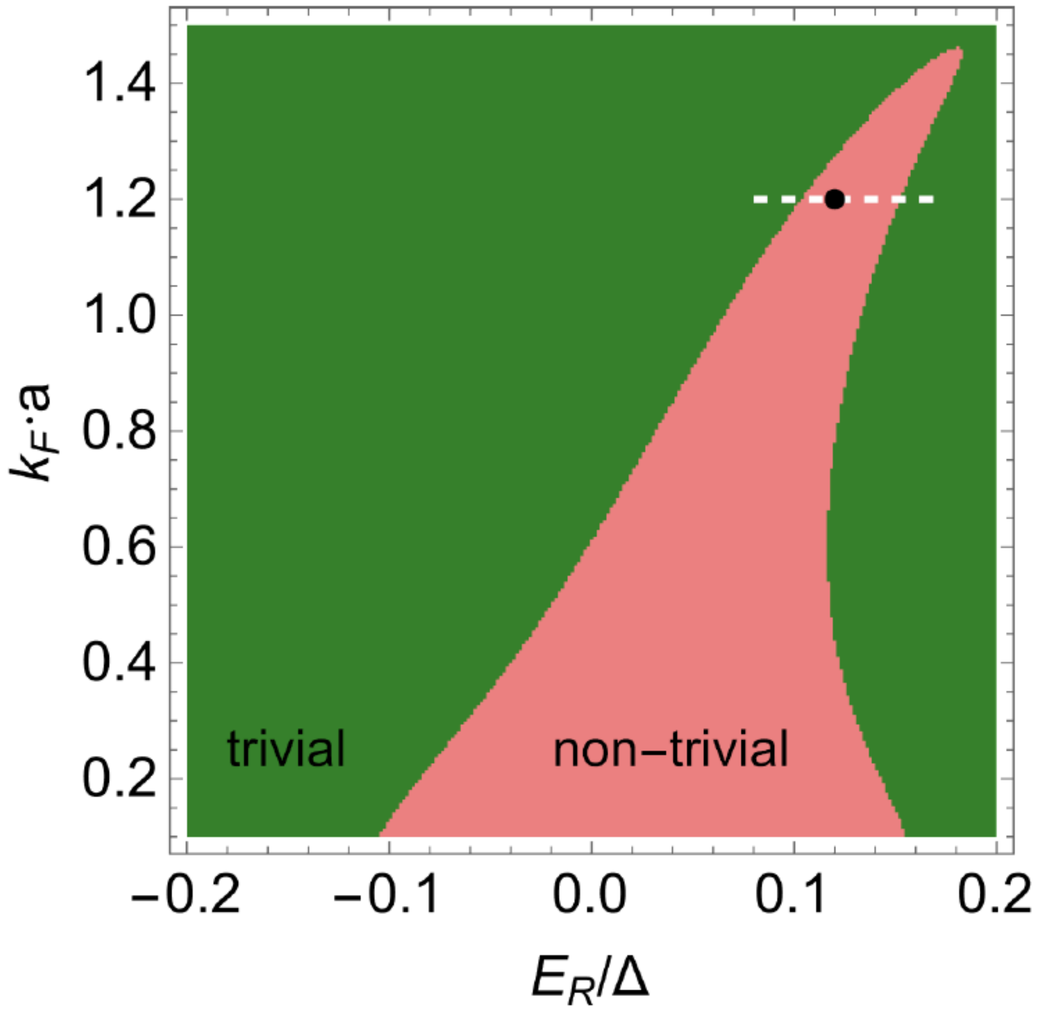
- Only consider coupling between closes corrals
- Further couplings negligibly small for geometric reasons



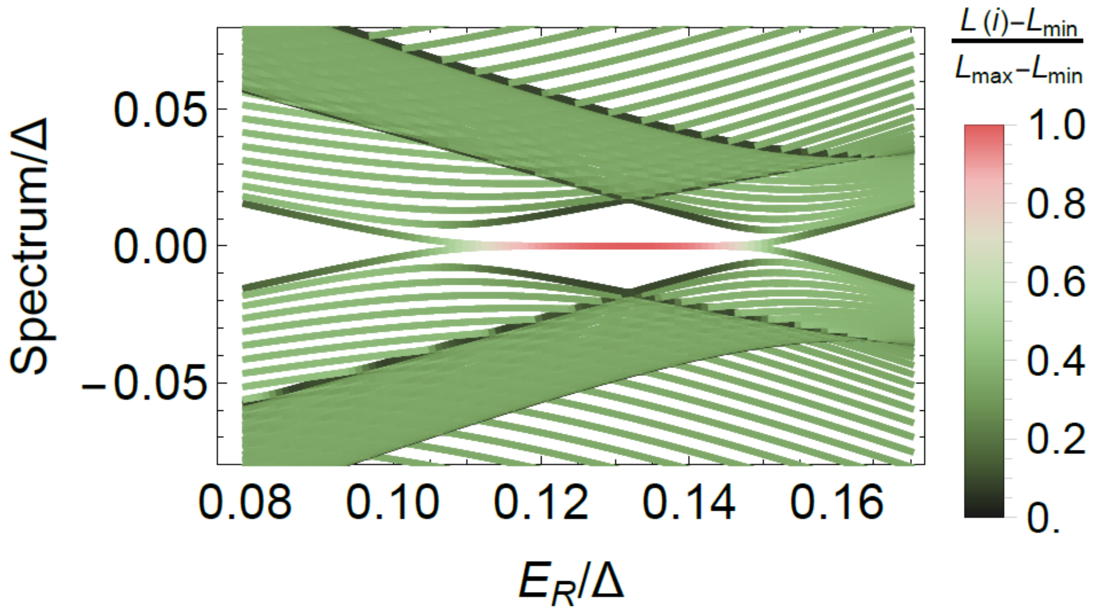


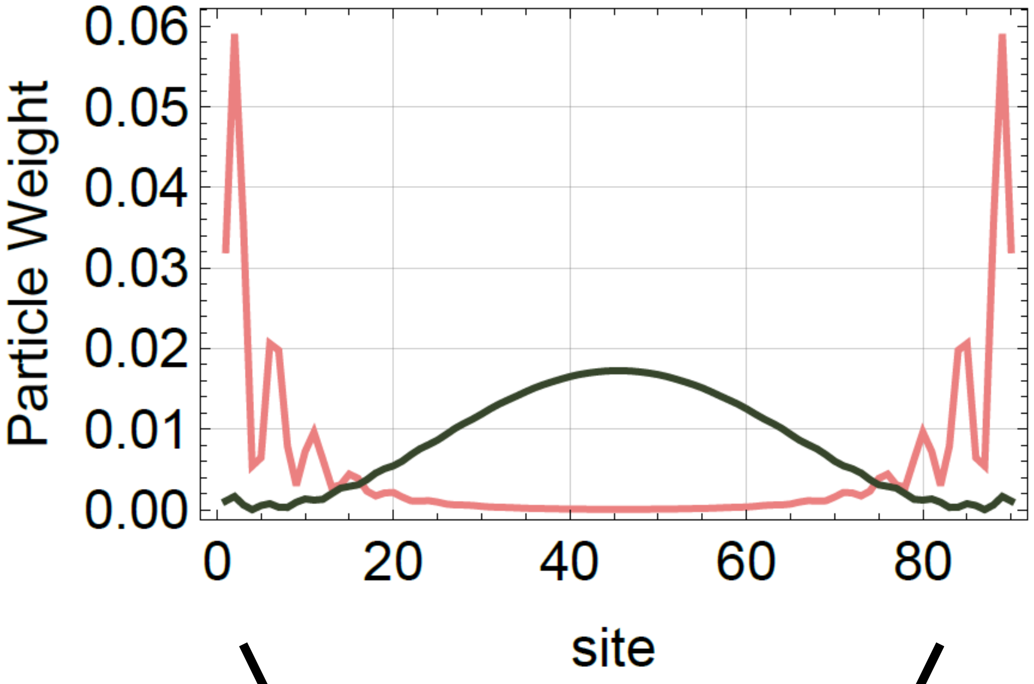
1D topological phase diagram



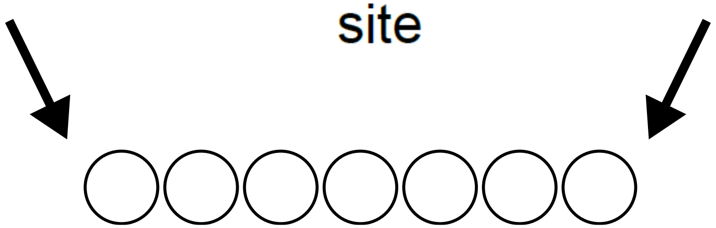


Majorana bound states



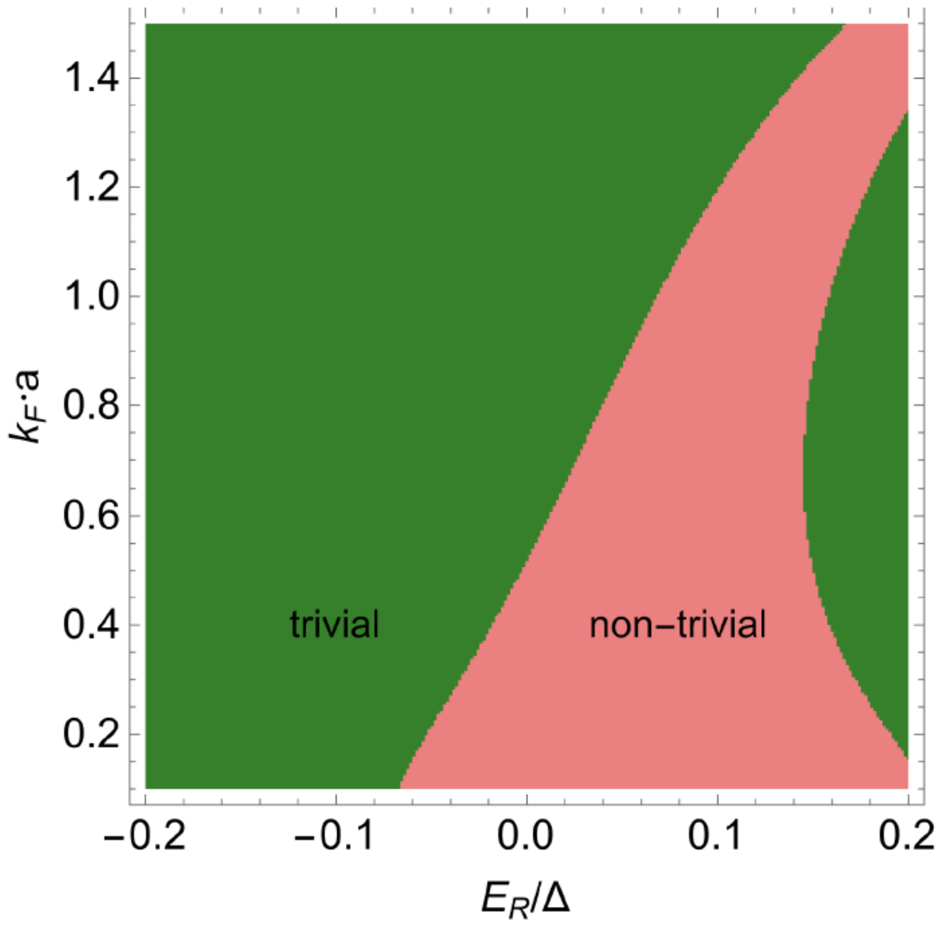


2x Kramers degenerate
Majorana boundary modes

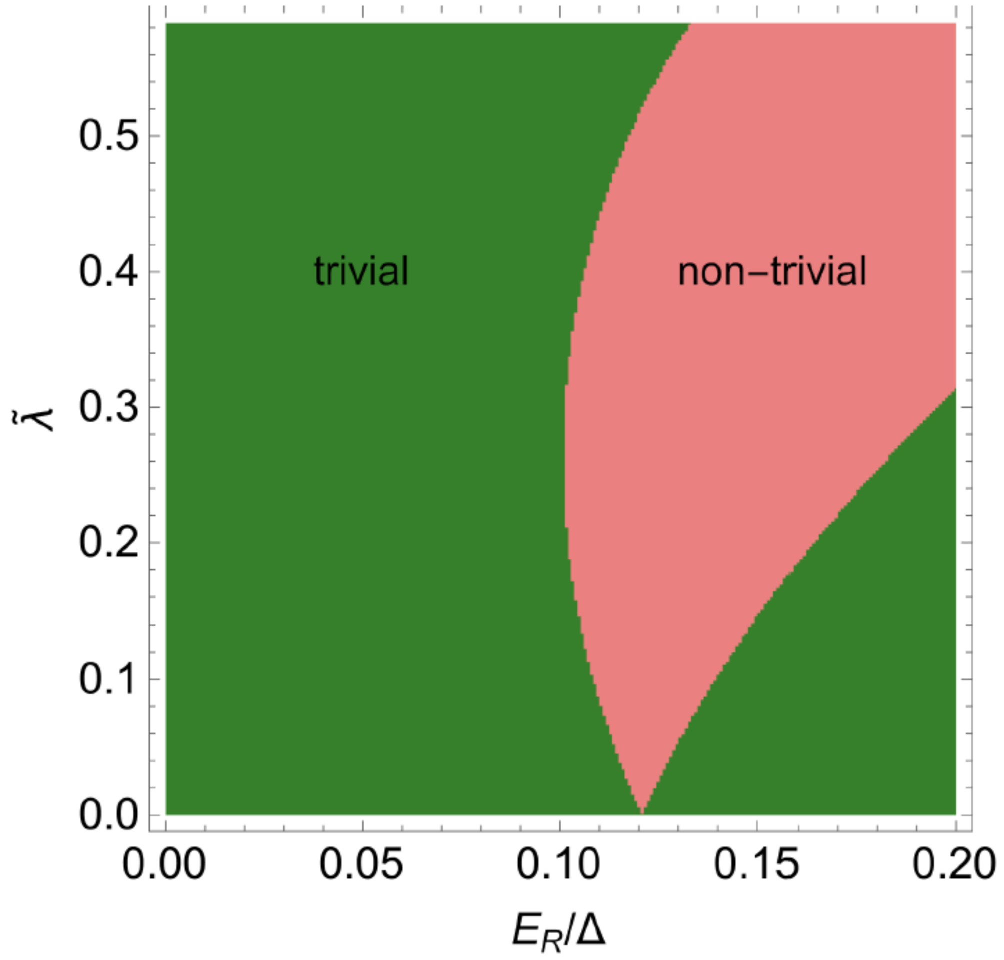


Ardonne, Budich Phys. Rev. **B** 88, 134523 (2013)
C.-H. Hsu, P. Stano, J. Klinovaja, and D. Loss, Phys. Rev. Lett. 121, 196801 (2018)

2D topological phase diagram

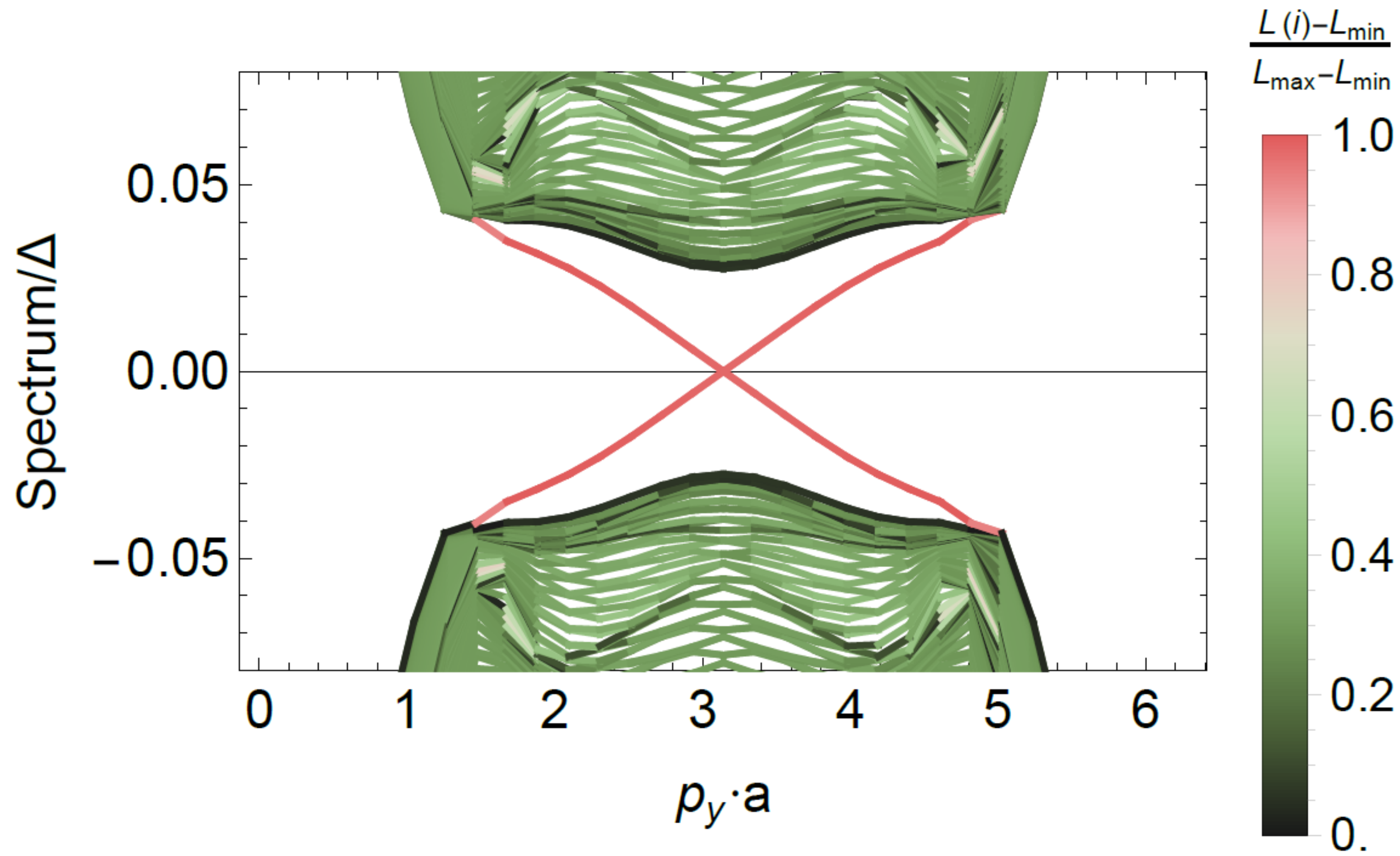


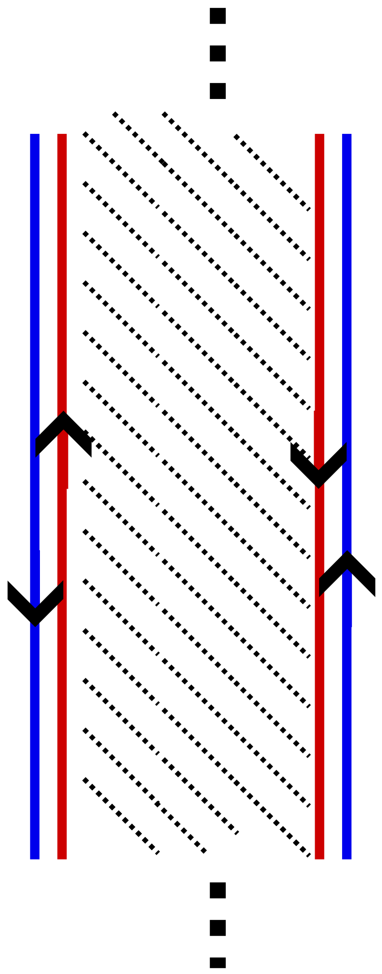
distance of impurities a and Fermi wave vector k_F



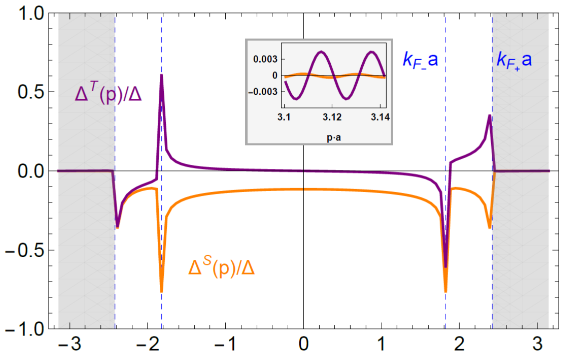
effective spin-orbit coupling

Chiral Majorana edge channels





Solution 2 – suppress singlet superconductivity



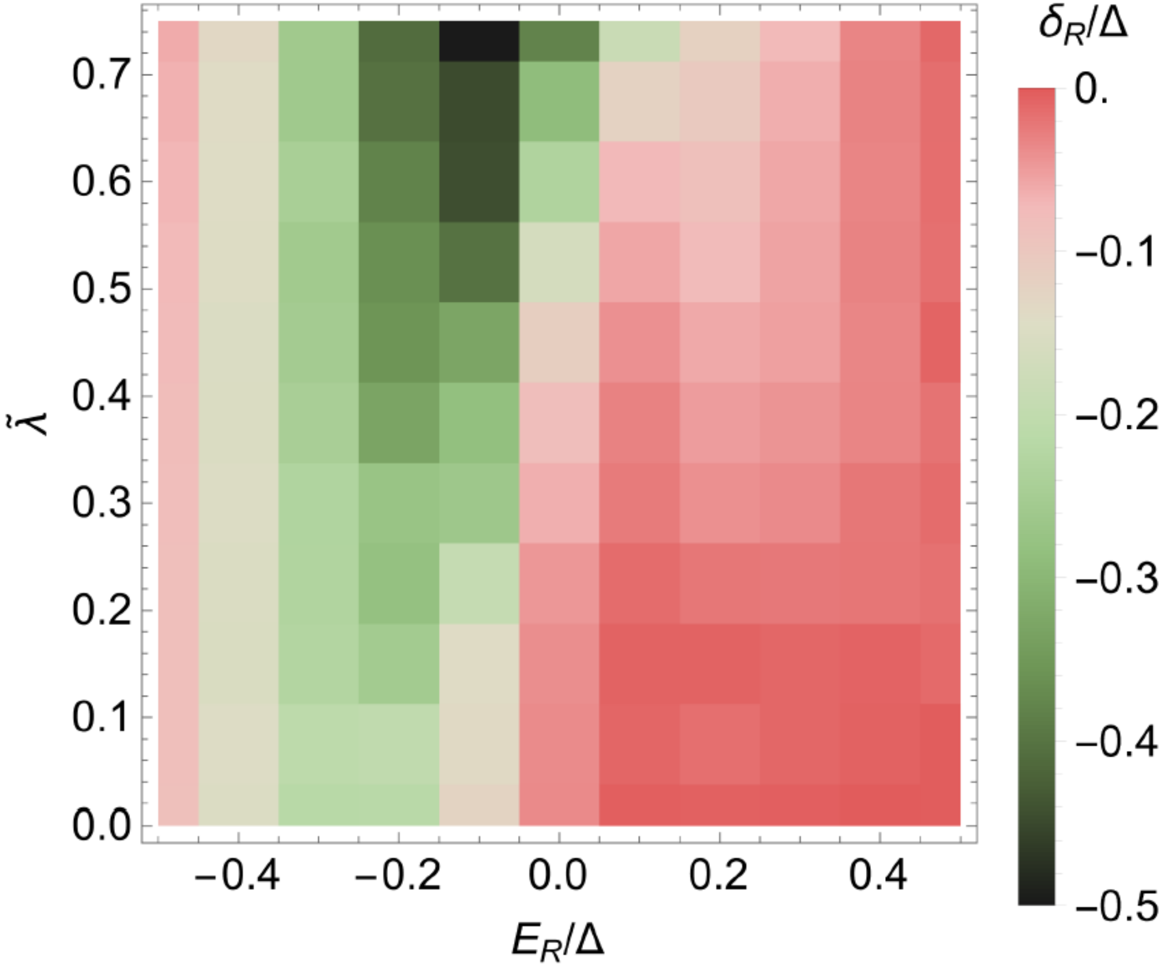
$$\hat{H}_{\mathrm{MS}} = E_{\mathrm{R}} \sum_{\sigma,j} d^{\dagger}_{\sigma,j} d_{\sigma,j}$$

$$+ U \sum_{j} d^{\dagger}_{\uparrow,j} d_{\uparrow,j} d^{\dagger}_{\downarrow,j} d_{\downarrow,j}$$

Mean field decoupling and minimization of free energy

$$\delta := \langle d_{\uparrow,j} d_{\downarrow,j} \rangle$$

$$\mathcal{F} = -T \int d\mathbf{k} \sum_{m} \ln\left(1 + e^{-\beta E_m(\mathbf{k})}\right) + U\delta^2$$



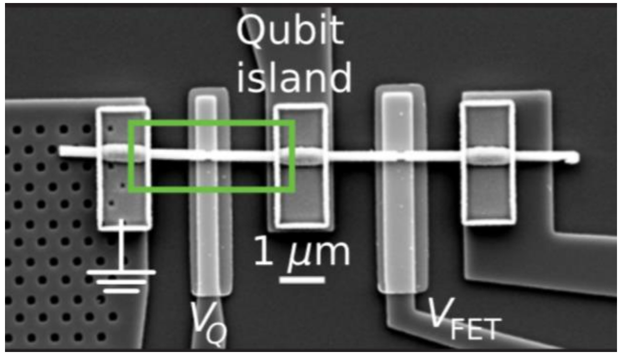
Applications of Machida-Shibata states

Kazushige Machida

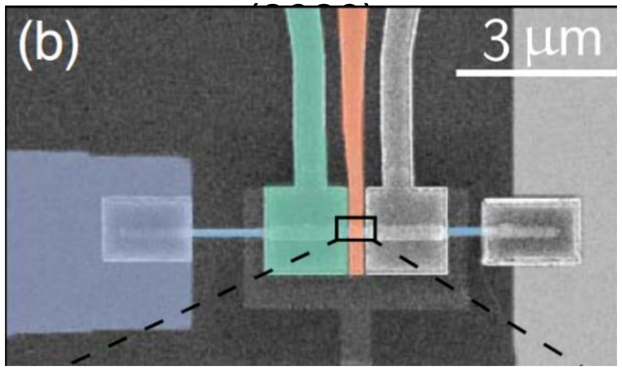
.. thank you for finding utility of Machida-Shibata lattice as a possible time-reversal symmetric topological superconductivity.
I really hope that your proposal is realized near future.

Sincerely,
Kazu Machida

Additional prospects:
Transmon qubit + quantum dot

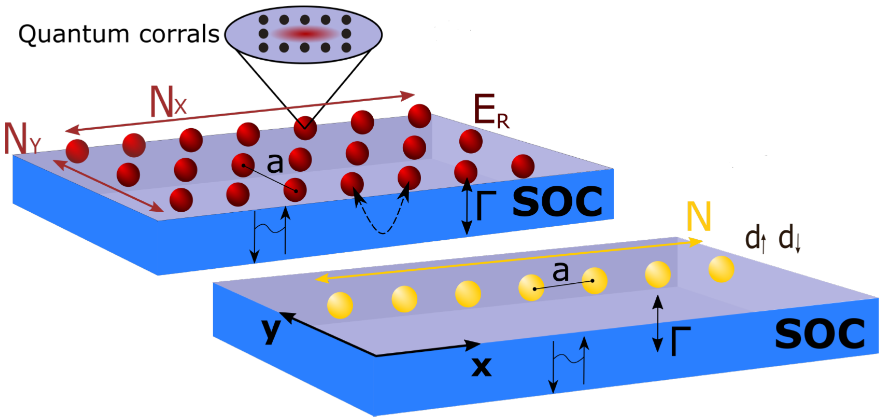


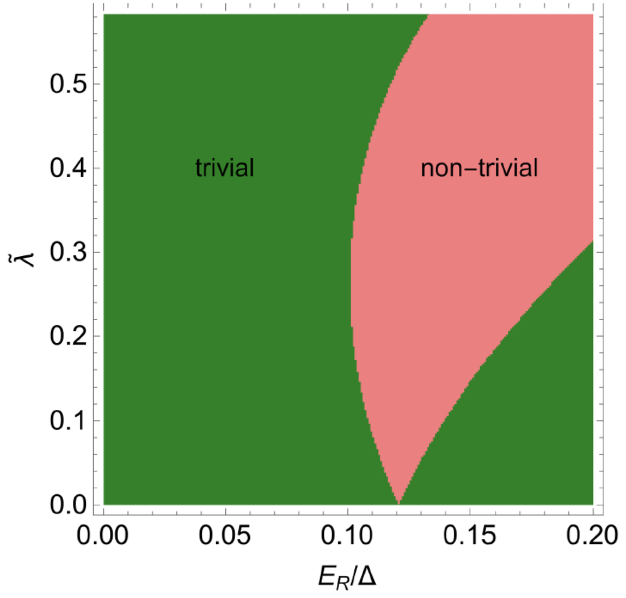
Kringhøj et al. PRL **124**, 246803

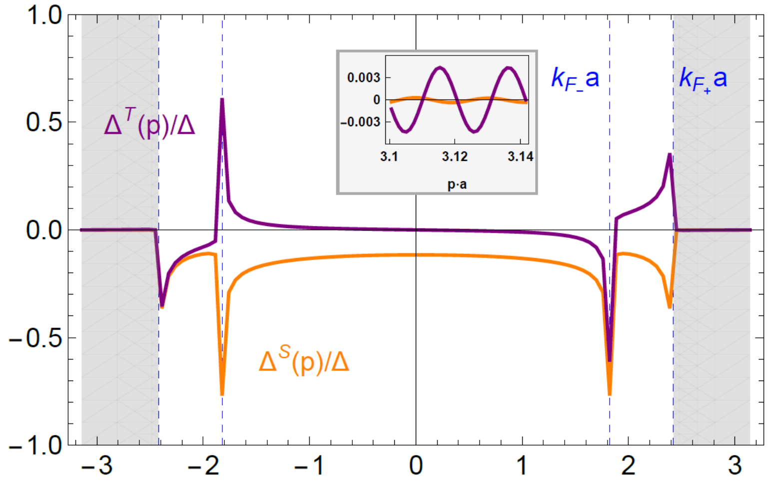


Bargerbos et al. PRL **124**, 246802 (2020)

Summary





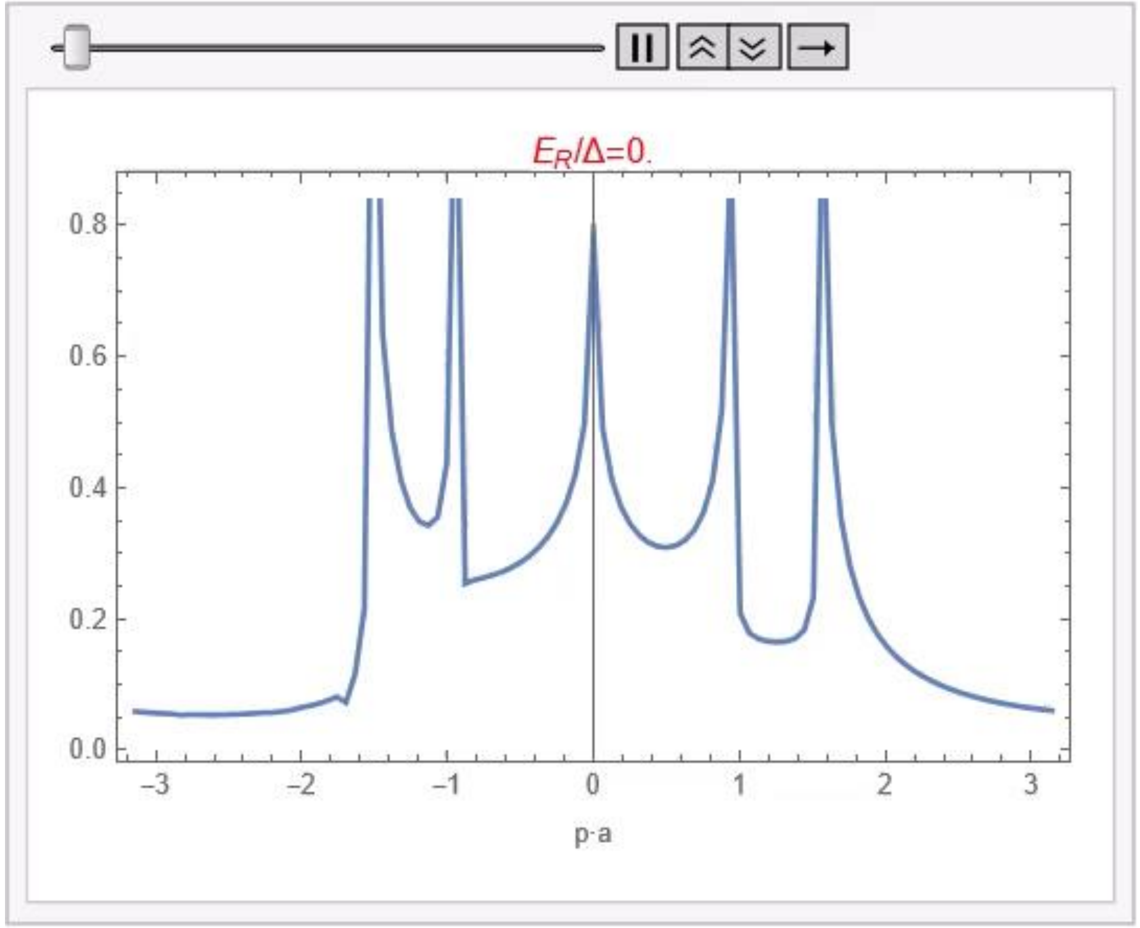


Machida-Shibata lattices

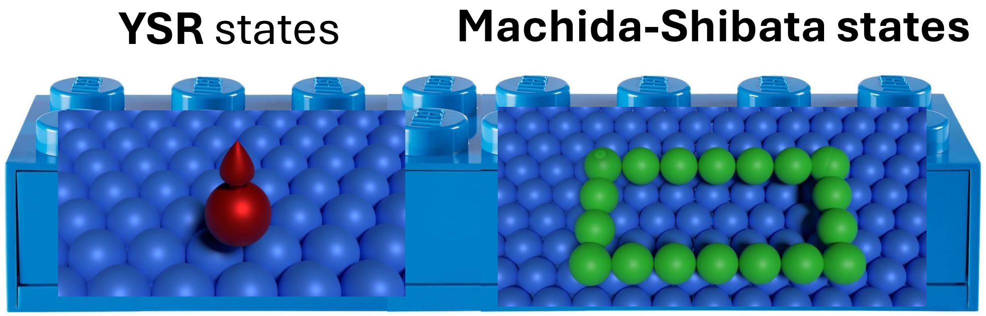
Topological superconductor,
class DIII,
Kramers degenerate Majorana modes

Compensating singlet and triplet superconductivity in momentum space

Gap closure and opening

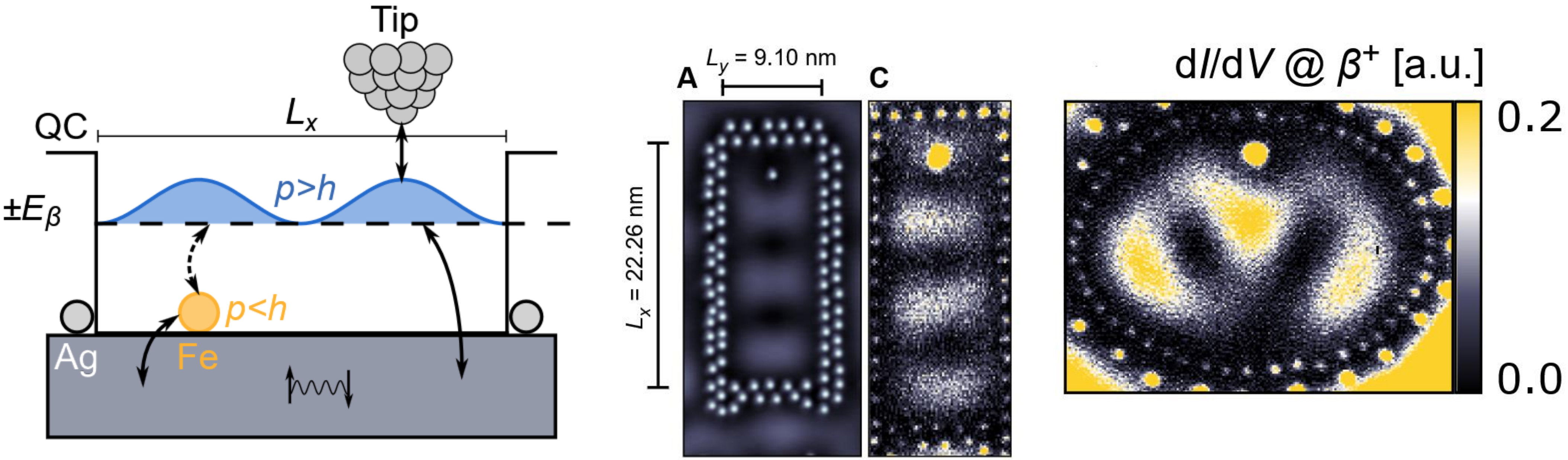


Outlook



- Hybridizing Machida-Shibata states and Yu-Shiba-Rusinov states

That, Xu, Ioannidis, Schneider, Posske, Wiesendanger, Morr, Wiebe, arXiv:2410.16054



- **Splitting Kramers degeneracy**, further topological phases upon hybridization

Acknowledgements

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Chang Xu (U Chicago)
Lucas Schneider
Khai Tôn That



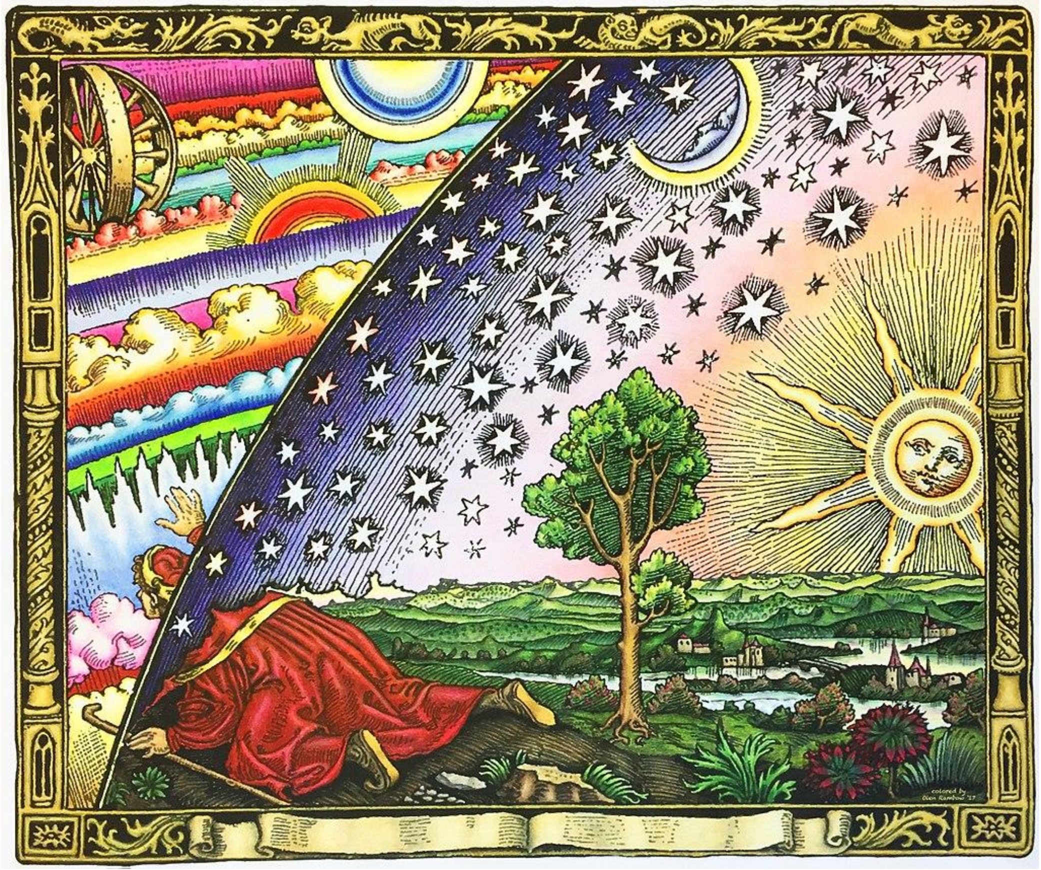



Thank you

erc Starting grant
QUANTWIST

DFG Deutsche Forschungsgemeinschaft

UHH EXZELLENZCLUSTER
CUI: ADVANCED
IMAGING OF MATTER



Flammarion engraving, Paris 1888
„Die Neugier hinter die Dinge zu schauen“