Topology in condensed matter physics
exercise sheet SSH lecture

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hand out: June 28, discussion: July 5

13 Warming up for the SSH model

As warm up for the lecture on the nonhermitian Su-Schrieffer-Heeger (SSH) model, consider the following questions about the hermitian SSH model. The questions refer to the textbook of Asbóth, L. Oroszlány, A. Pályi, chapter 1. You can also use the lecture videos and notes as a reminder.

1. (1 point) Write down the Hamiltonian of the SSH model (see Eq. 1.1 of the textbook) on a real-space basis, for a chain of $N = 4$ unit cells.
2. (1 point) Diagonalize the Hamiltonian for intra unit cell coupling $v = 0$ and extra unit cell coupling $w = 1$. Write down the eigenvectors and eigenenergies.
3. (1 point) Are there any edge states? Write down their eigenvalues and eigenvectors.
4. (1 point) Show that edge states have support on one sublattice only, while bulk states have equal support on both sublattices. As an example, consider both edge states and one bulk state.
5. (1 point) Explicitely Fourier transform Eq. 1.6 and derive step by step Eq. 1.14.
6. (1 point) Using Eq. 1.38, calculate explicitely the winding number for $v = 0$ and $w = 1$.
7. (1 point) Diagonalize the 2 by 2 Hamiltonian in k-space for $v = 0$ and $w = 1$. Plot the dispersion. Why do you have only two bands? Why are the bands flat?
8. (1 point) Diagonalize the Hamiltonian of the SSH model (see Eq. 1.1 of the textbook) on a real-space basis, for a chain of N = 4 unit cells, setting v = 0.05 and w = 1. Write down the eigenvectors and eigenenergies of the edge states.
9. (1 point) Plot the wavefunction amplitudes of these edge states as function of the lattice sites. What do you observe comparing them with the ones obtained in 3)? How does their spatial decay look like? (Hint: take a look at the section 1.5.2 of the textbook). How can you explain this difference physically?
10. (1 point) Integrate explicitely the winding number in Eq. 1.38 for general values of v and w. Comment on your result.

End