(1) # Lecture II

last lecture: Quantum mechanics cheat sheet !
classical computing } gates, (qu)bits,
quantum computing } completeness

questions from last time:

- online talks/exams possible upon reasonable request
- Topics assigned at end of this week (nuudel at latest on Wednesday)
- Do in-between measurements make quantum computing more powerful?
  → There are both, complete unitary & measurement-based schemes → Ref. put online about measurement based QC
- Video of last lecture has small video area. Can somebody edit the video?

Assign to projects online!

2 projects

Further questions about last lecture?

②

2. Quantum annealing

Physical principles: Temperature, adiabatic time evolution

optimization problems

quantum annealers & how to control them

2.1. Ground state & Temperature

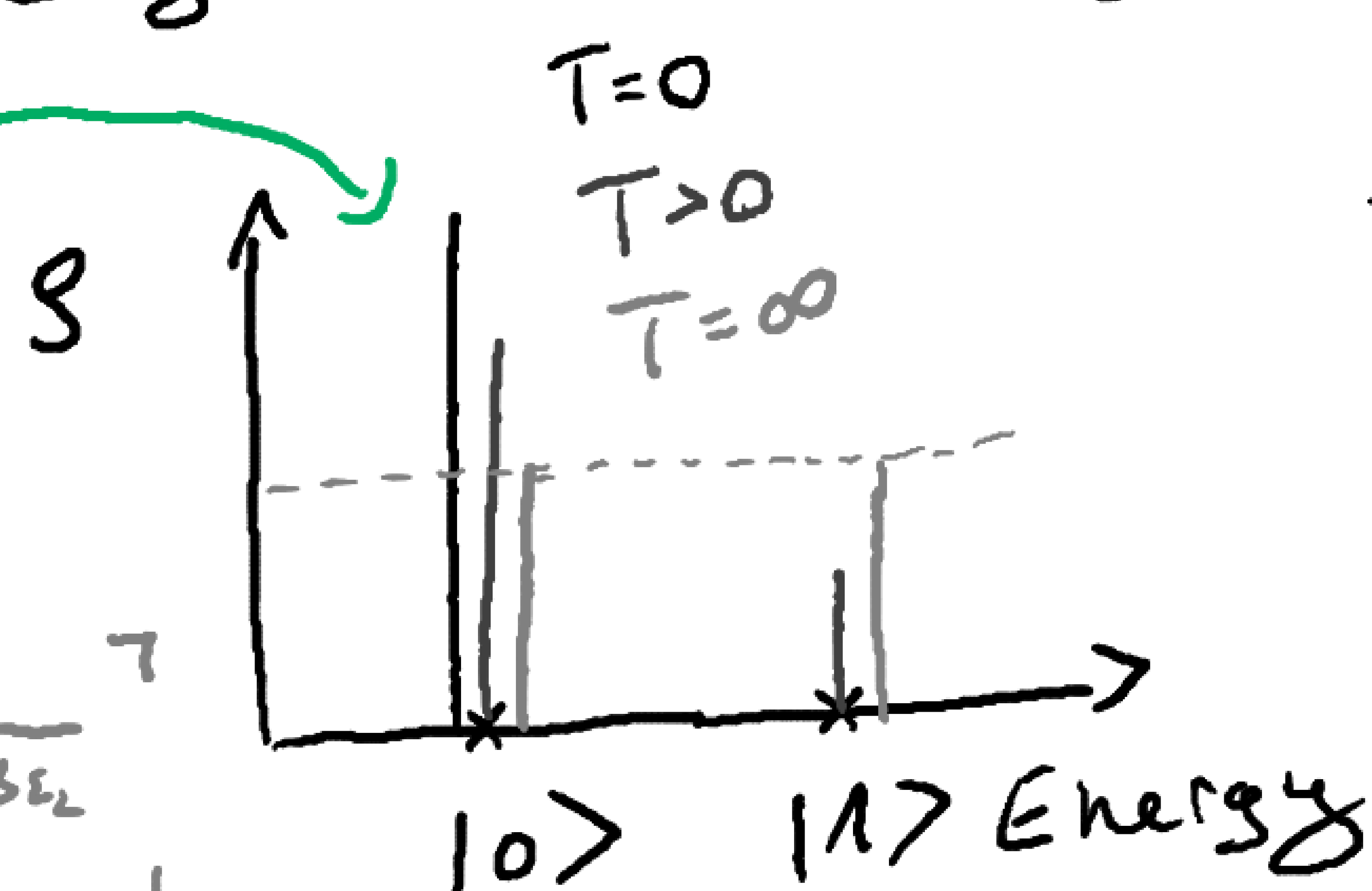
Hamiltonian $\mathcal{H}$ (linear operater on Hilbert space)

↳ Eigenvectors: States with well-defined energy

↳ Eigenvalues: Energies of respective eigenstate

ground state highest probability

$$\left[\rho_i = \frac{e^{-\beta \varepsilon_i}}{e^{-\beta \varepsilon_1} + e^{-\beta \varepsilon_2}} \right]$$



Temperature: Chance to find system in state $|i\rangle$ is $\rho_i \propto e^{-\beta \varepsilon_i}$

(Boltzmann factor) $\beta = \frac{1}{k_B T}$

Wake-up brain:

$$\boxed{\text{When measuring a system it most likely is in ground state}}\ ?$$

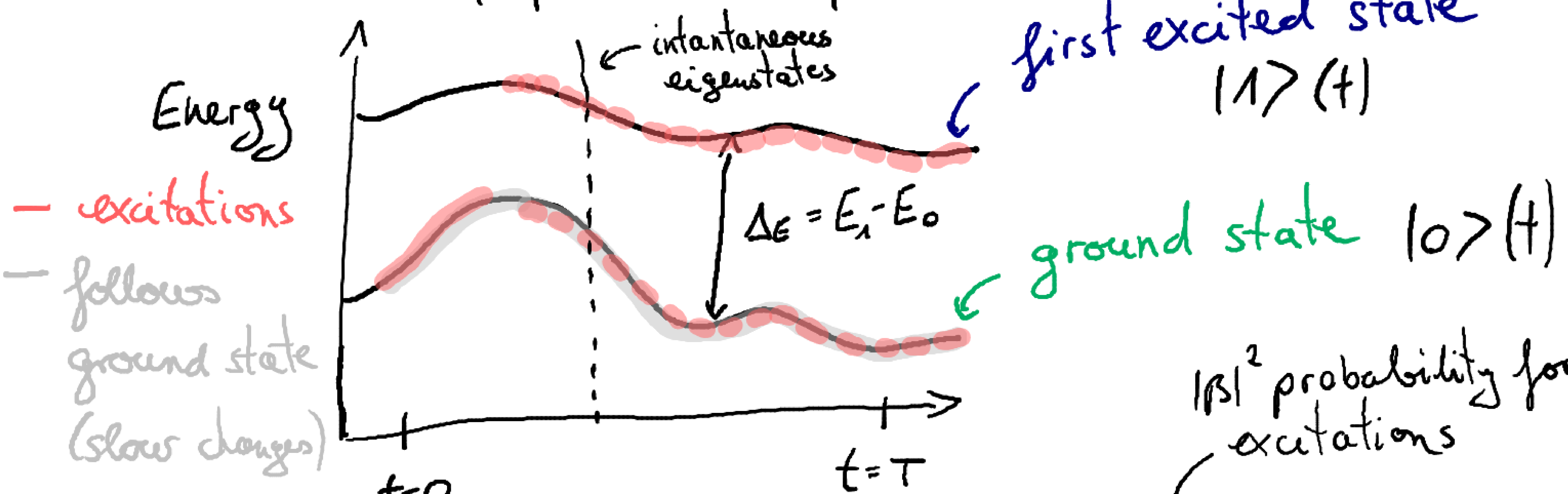
↳ No → $\rho_{GS} < \sum \rho_{\text{not ground state}}$ for very small T in macroscopic systems

③

2.2. The adiabatic theorem of quantum mechanics

· Schrödinger equation

$$i\hbar\,\partial_t|\psi\rangle = \mathcal{H}(t)\,|\psi\rangle$$



In general

$$|\psi\rangle(t=0) = |0\rangle, \quad |\psi\rangle(T) = \alpha\,|0\rangle + \beta\,|1\rangle$$

Adiabatic theorem (parts of it)

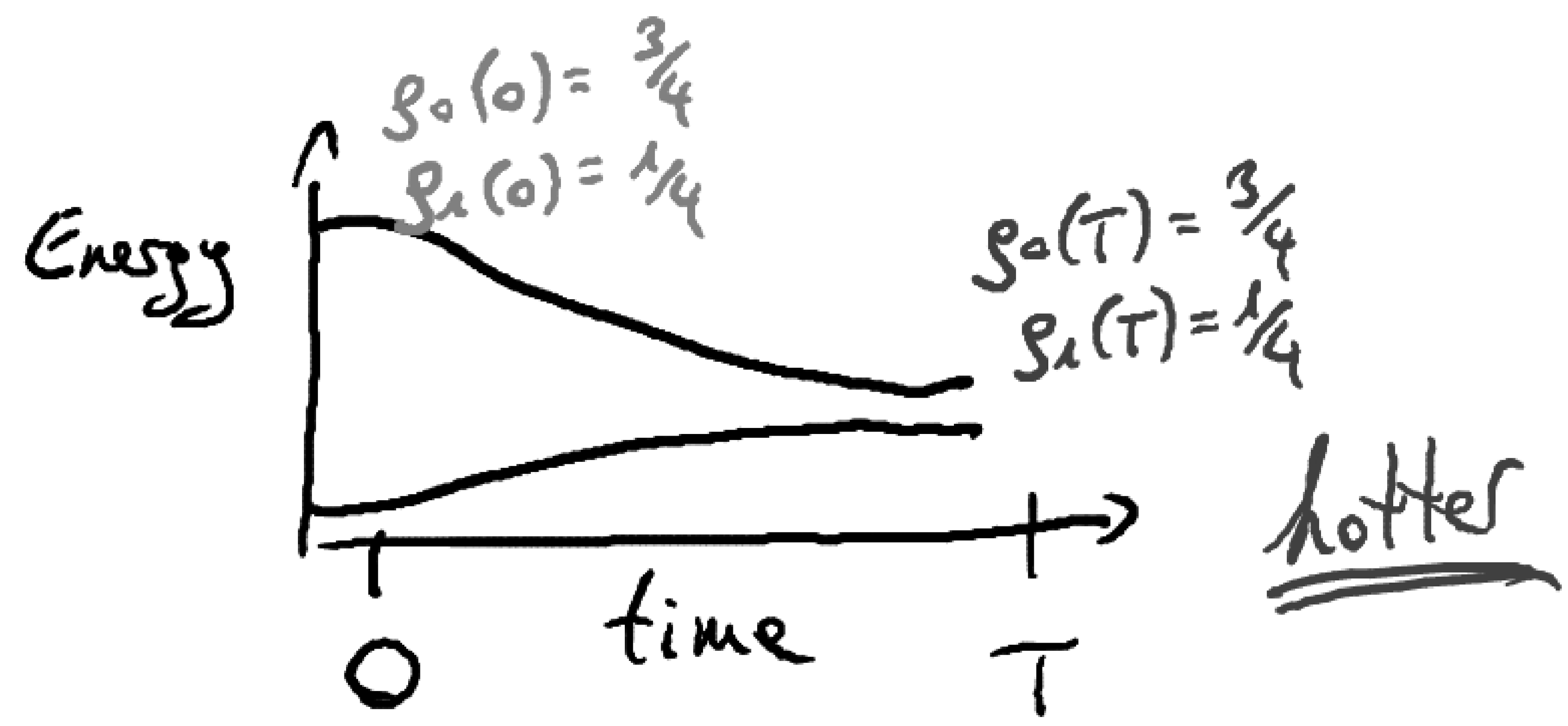
proof at posske.de → lectures → Top. in cond. matt. '16 lectures 11 & 12

If a system $\mathcal{H}(t)$ is manipulated sufficiently slowly [such that $\mathcal{H}'(t) \ll \frac{\text{level spacing}}{T}$], the initial eigenstates evolve to the instantaneous eigenstates they are (smoothly) connected to. [changes too slowly for excitations]

In particular: The ground state of the initial Hamiltonian evolves to the ground state of the final one

④

Remark: Nomenclature 'adiabatic' means 'no heat exchange'. But system can still be heated or cooled

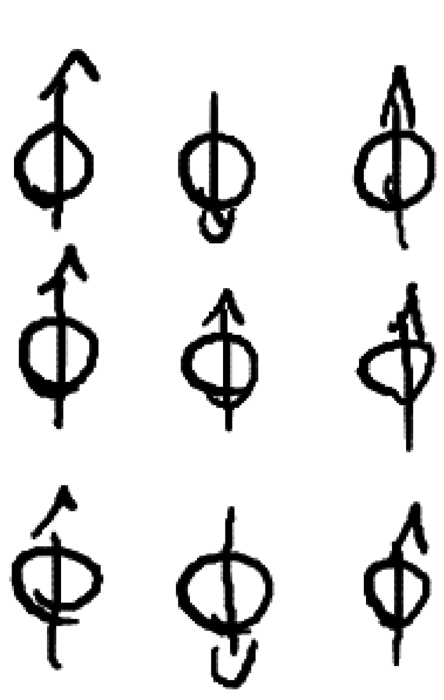


Entropy is conserved.

2.3. Quadratic optimization problems

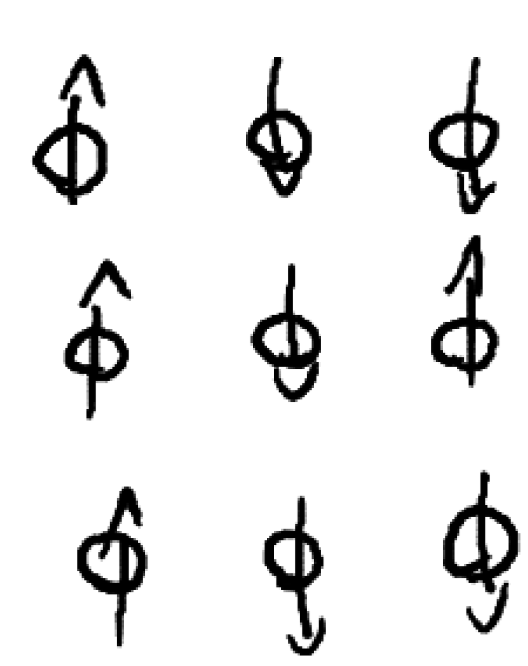
The Ising model [Ernst Ising 1900-1998
1922-1924 Promotion bei
Wilhelm Lenz

classical



'Spins'
$S_j \in \{1,-1\}$
$\equiv \{\uparrow, \downarrow\}$

quantum



spins

$\left(\mathrm{span}\{|\uparrow\rangle, |\downarrow\rangle\}\right)^n$

$S_j = \frac{\hbar}{2}\begin{pmatrix}\sigma_x \\ \sigma_y \\ \sigma_z\end{pmatrix}$

$$\mathcal{H} = \sum_j B_j S_j^z + \sum_{jk} J^{kj} S_j^z S_k^z$$

↳ external magnetic field

5

Question: Given external fields $\{B_j\}$ & couplings $\{J^{jk}\}$, what is the ground state of the system?

Answer: Hard to find [only easy if S^z continuous]

↳ · Decades of mathematical theories & algorithms: Quadratic optimization

· hundreds of Professorships

Liebevoll

Qubo: quadratic binary optimization

Remark: (1) Quantum & classical system have the same ground state (if not degenerate)

[Because Hamiltonian is diagonal matrix

$$\sigma^z \otimes \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(2) The ground state can be measured non-destructively and locally because $[\mathcal{H}, S_j^z] = 0$

[$\mathcal{H}\left[S^z|GS\rangle\right] = S^z \varepsilon |GS\rangle = \varepsilon S^z|GS\rangle$

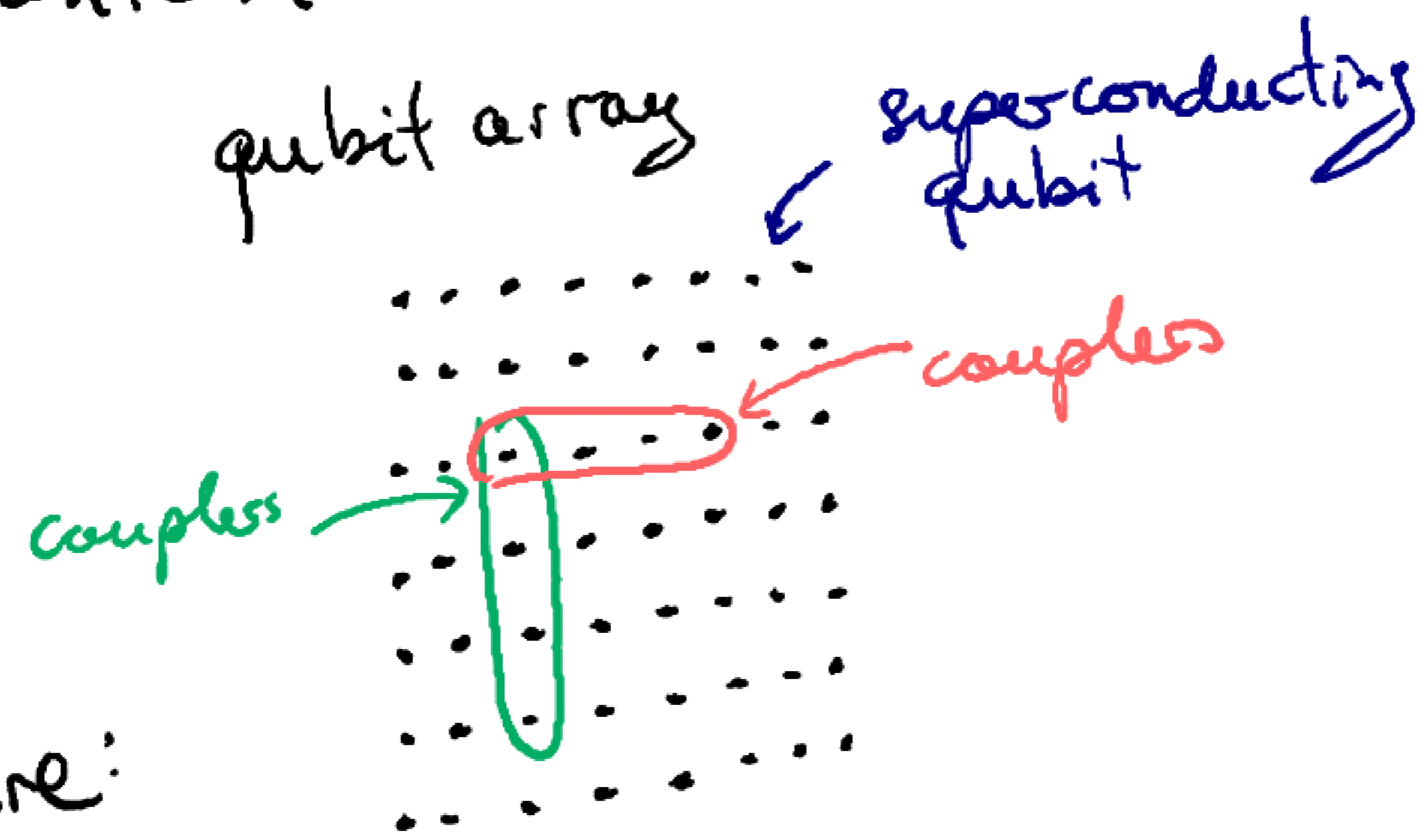
$\Rightarrow$ $S^z|GS\rangle$ is eigenstate of $\mathcal{H}$ with energy ε $\Rightarrow$ $S^z|GS\rangle$ is also ground state]

(3) A large class of problems can be mapped to Qubo problems

- constrained binary/integer problems

6 2.4. Quantum annealers

Quantum shortcut to Quadratic optimization?



e.g.
D WAVE
quantum annealers

procedure:

1. Initial state: superposition of all possible product basis states

$$\psi_0 = \left(|\uparrow\rangle + |\downarrow\rangle\right)^N$$

from initial Hamiltonian

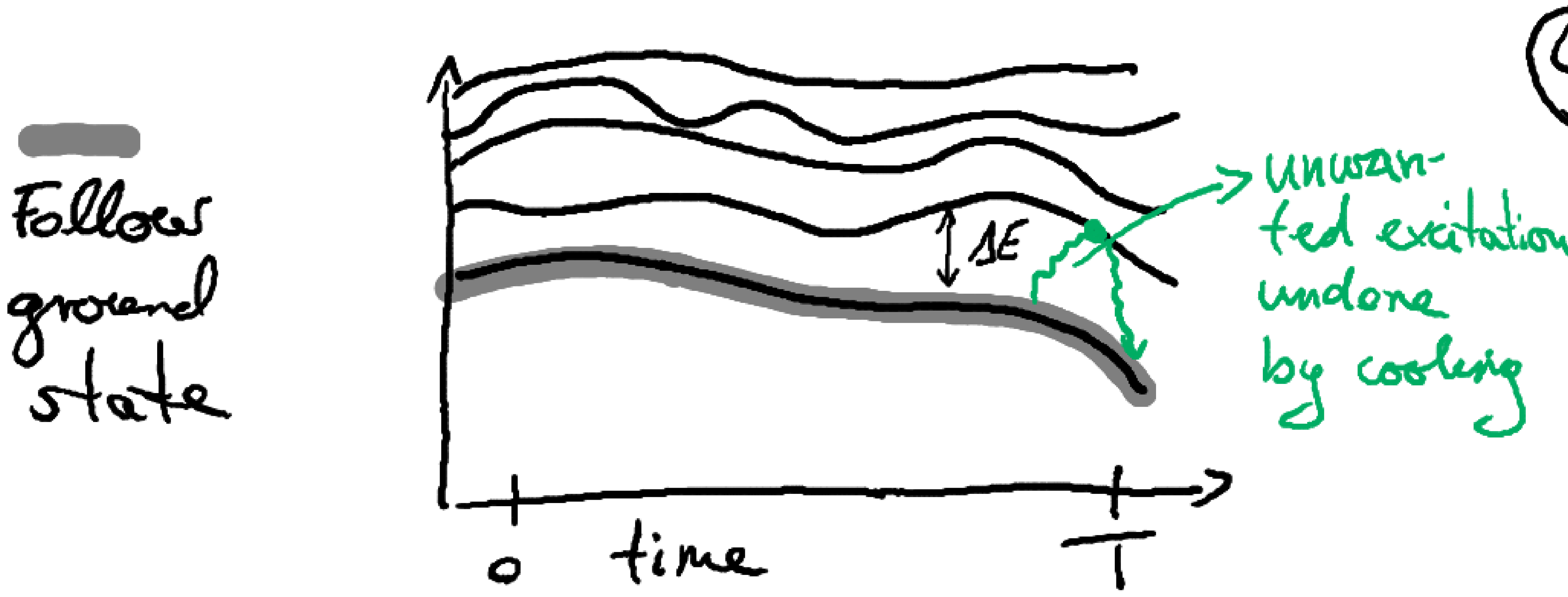
$$\mathcal{H}_0 = \sum J_0 S^x_j S^x_i$$

construct Qubo: $\mathcal{H}_Q = \sum_j B_j S^z_j + \sum_{kl} J^{kl} S^z_k S^z_l$

2. Construct time-dependent Hamiltonian

$$\mathcal{H}(t) = \left(1 - t/T\right) \mathcal{H}_0 + t\,\mathcal{H}_Q$$

3. Let time-evolution run sufficiently slowly $H'(t) \ll T\Delta E$



4. Read out final state as solution to optimization problem

Remarks:

7

- Permanent cooling helps undoing accidental excitations. Attention: Neither this nor tunneling is the (main) working principle of quantum annealing
- Procedure is repeated thousands of times, best run is taken as solution
- No (known) way to determine needed time or verify solution
- No principle advantage against classical architectures

2.5. Controlling a quantum annealer

DWAVE Ocean SDK

Leap → online programming of quantum annealer

Example I Quadratic binary optimization problem

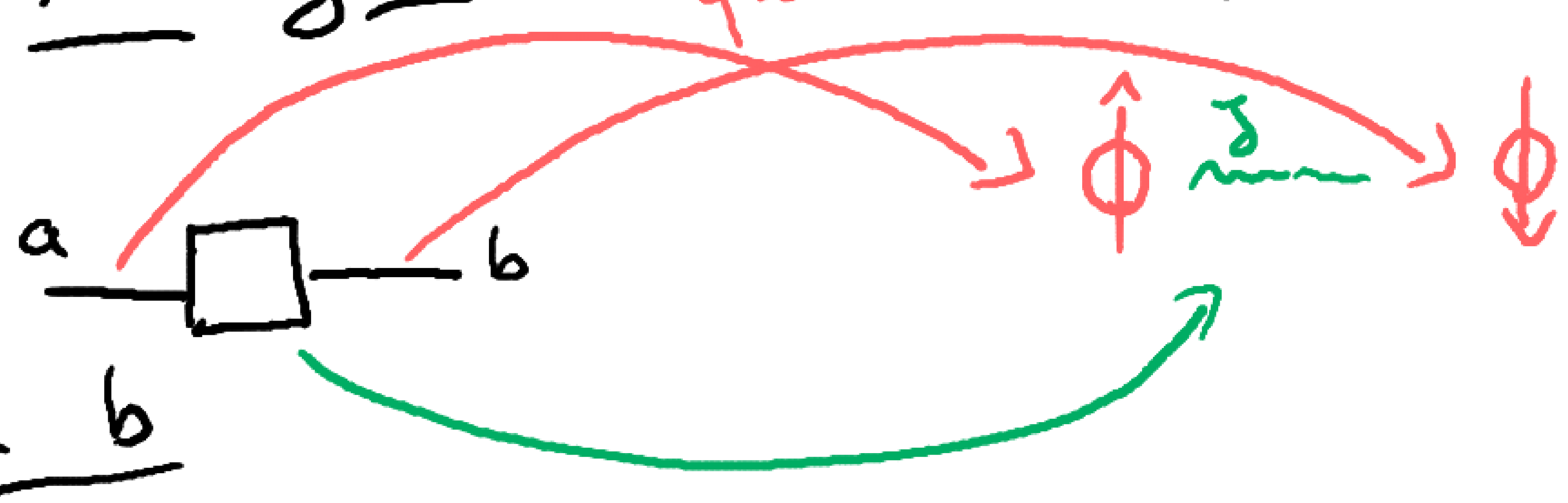
↳ see online ressources

Who thinks Qubos are dull?
Who feels inspired by Qubos already?

Example II / Inspiration: binary circuits

(8) classical gates can be translated to the Ising model by constructing degenerate ground states that attach desired output to input.

NOT gate



$$\mathcal{H} = S_1^z S_2^z \cdot J$$

a	b
1	0
0	1

coupling

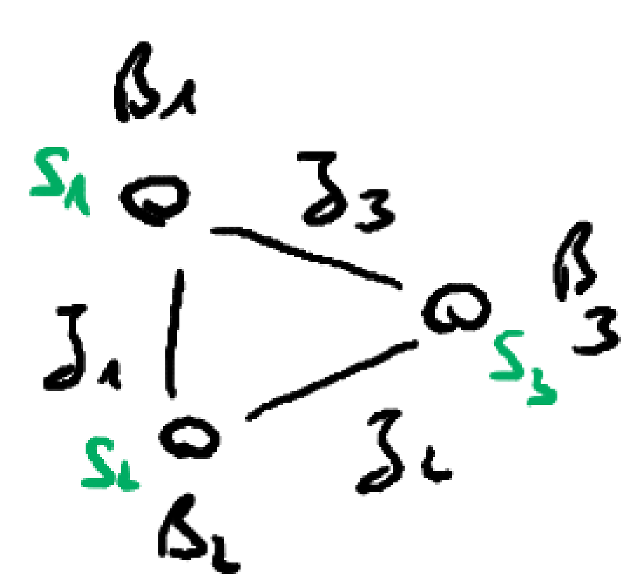
encode as ground state

energies/J

	↑↑	J
$1 \to 0$	↑↓	$-J$
$0 \to 1$	↓↑	$-J$
	↓↓	J

$J > 0$

AND Gate



$$\mathcal{H} = \sum_i \underbrace{B_i S_i^z}_{\text{magnetic energy } E_M} + \underbrace{J_i S_{i+1} S_{i-1}}_{\text{coupling energy } E_C}$$

periodic

a_1 a_2	b
1 1	1
0 1	0
1 0	0
0 0	0

ground states

S_1	S_2	S_3	E_M	E_C	total
1	1	1	0	$-\alpha$	$-\alpha$
1	1	-1	$-\alpha$	2α	α
1	-1	1	$\alpha/2$	$-\alpha/2$	0
1	-1	-1	$-\alpha/2$	$-\alpha/2$	$-\alpha$
-1	1	1	$\alpha/2$	$-\alpha/2$	0
-1	1	-1	$-\alpha/2$	$-\alpha/2$	$-\alpha$
-1	-1	1	α	2α	3α
-1	-1	-1	0	$-\alpha$	$-\alpha$

Solution: ($1 \leftrightarrow 2$ symmetry)

$B_1 = -\frac{1}{4}\alpha \quad J_1 = -\frac{3}{4}\alpha$

$B_2 = -\frac{1}{4}\alpha \quad J_2 = -\frac{3}{4}\alpha$

$B_3 = \frac{1}{2}\alpha \quad J_3 = \frac{1}{2}\alpha$

with $\alpha > 0$

If $S_i \to -S_i \Rightarrow E_M \to -E_M$, $E_C \to E_C$

$\Rightarrow$ Qubo is 'computational complete'

By setting input $\Rightarrow$ compute output as solution to optimazation problem.

By setting output $\Rightarrow$ compute fitting input " " " " ".

⑨ Consequences: Solving Qubo helps solving hard tasks if their inverse is easy

- multiplication ↔ factorization
- encrytion ↔ decrytion

...

Online demonstration factorization

↳ Leap

! Word of caution → Youtube
'Sabine Hossenfelder, quantum hype'
https://youtu.be/6-aGlvUomTA

Good luck with your projects (assigned at end of week)

Contact us if problems arise

Supervisors

T → Thore Posske tposske@physnet...
F → Felix Gerken fgerken@physnet...
G → Giannis Ioannidis iioannid@physnet...

Vote about deadline & final talks online

↳ participate